

8º Congresso do Comité Português da URSI

"Drones e veículos autónomos: desafios do presente e do futuro"

Lisboa, Portugal, 28 de Novembro, 2014

INSTITUIÇÕES ASSOCIADAS



UNIVERSIDADE DE COIMBRA



ASYMMETRIC BAND DIAGRAMS IN PHOTONIC CRYSTALS WITH A SPONTANEOUS NONRECIPROCAL RESPONSE

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Motivation

Tellegen media (nonreciprocal achiral bi-isotropic media)

Constitutive relations

$$\mathbf{D} = \epsilon_0 \epsilon \mathbf{E} + c^{-1} \kappa \mathbf{H}$$

$$\mathbf{B} = \mu_0 \mu \mathbf{H} + c^{-1} \kappa \mathbf{E}$$

$\kappa \rightarrow$ Nonreciprocity
parameter

Motivation (cont.)

The primary E and B fields in an unbounded Tellegen medium are undistinguishable from the primary fields in a equivalent simple medium

$$\begin{aligned} n & \quad (\mathbf{E}, \mathbf{B}) \\ & (\epsilon, \mu, \kappa) \end{aligned}$$

Tellegen medium

$$\begin{aligned} \mathbf{D} &= \epsilon_0 \epsilon \mathbf{E} + c^{-1} \kappa \mathbf{H} \\ \mathbf{B} &= \mu_0 \mu \mathbf{H} + c^{-1} \kappa \mathbf{E} \end{aligned}$$

Maxwell equations

$$\begin{aligned} \nabla \times \mathbf{E} &= i\omega \mathbf{B} \\ \nabla \times \mathbf{H} &= -i\omega \mathbf{D} \end{aligned}$$

$$\begin{aligned} n_d & \quad (\mathbf{E}, \mathbf{B}) \\ & (\epsilon_d, \mu_d) \end{aligned}$$

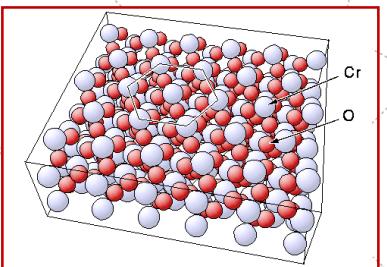
Simple isotropic medium

$$\begin{aligned} \mathbf{D}_d &= \epsilon_0 \epsilon_d \mathbf{E} \\ \mathbf{B} &= \mu_0 \mu_d \mathbf{H}_d \end{aligned}$$

$$\mu_d = \mu$$

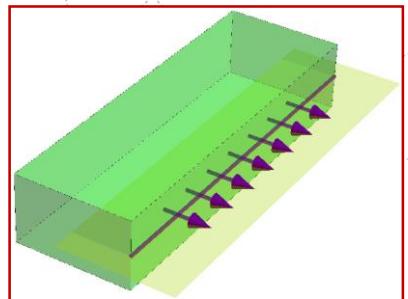
$$\epsilon_d = \frac{n^2}{\mu}$$

Motivation (cont.)



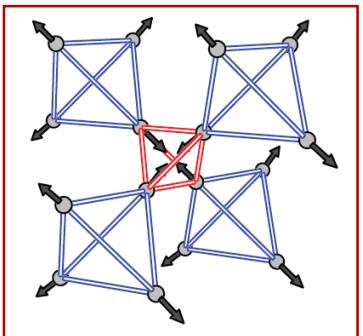
The chromium oxide Cr_2O_3 is an uniaxial Tellegen material

B. B. Krichevskov, V. V. Pavlov and V. N. Gridnev, *J. Phys.: Condens. Matter*, vol. 5, pp. 8233-8244, 1993.



Topological insulators may have a Tellegen-type electromagnetic response

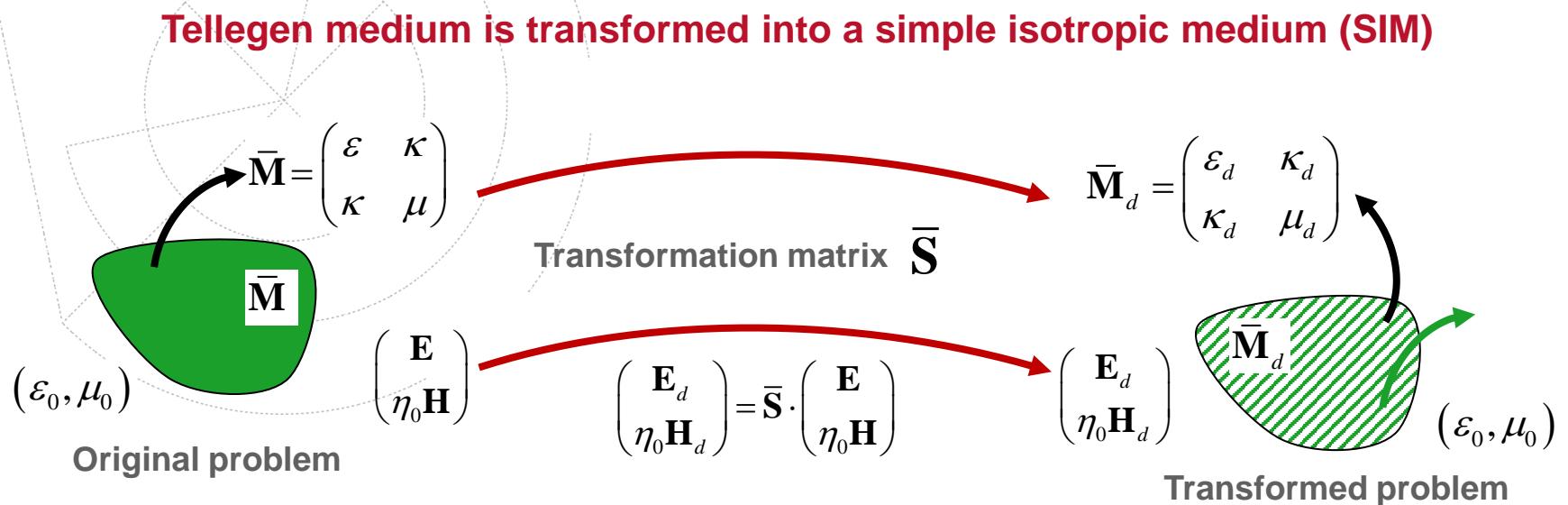
X. L. Qi, R. Li, J. Zang and S. C. Zhang, *Science*, vol. 323, pp. 1184-1187, 2009.



A Tellegen type response is allowed in particular crystals with a magnetic order

S. Coh and D. Vanderbilt, *Phys. Rev. B*, vol. 88, pp. 121106, 2013.

Duality theory



A. N. Serdyukov, A. H. Sihvola, S. A. Tretyakov and I. V. Semchenko,
Electromagnetics, vol. 16, no. 1, pp. 51-61, 1996.

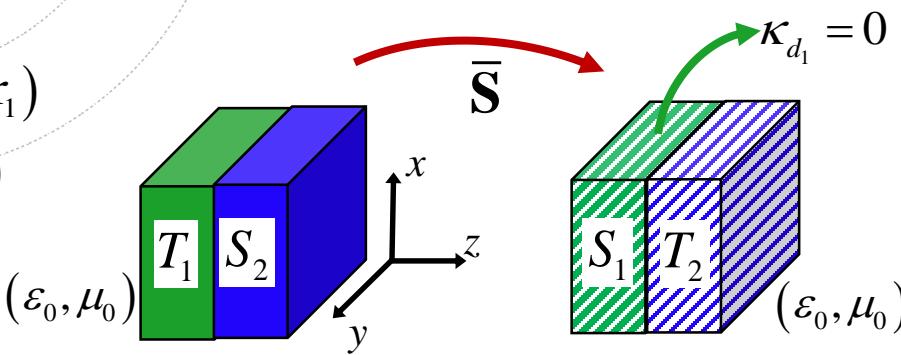
A. Lakhtakia and W. S. Weiglhofer, Electromagnetics, vol. 17, no. 2, pp. 199-204,
1997.

Duality theory (cont.)

Multi-material structures may **not** be reduced to simple structures

$$T_1 \rightarrow (\varepsilon_1, \mu_1, \kappa_1)$$

$$S_2 \rightarrow (\varepsilon_2, \mu_2)$$



$$S_1 \rightarrow (\varepsilon_{d_1}, \mu_{d_1})$$

$$T_2 \rightarrow (\varepsilon_{d_2}, \mu_{d_2}, \kappa_{d_2})$$

Objective

- Study in which **conditions** Tellegen structures are reducible to simpler structures
- Derive the **duality transformation** that enables such a reduction

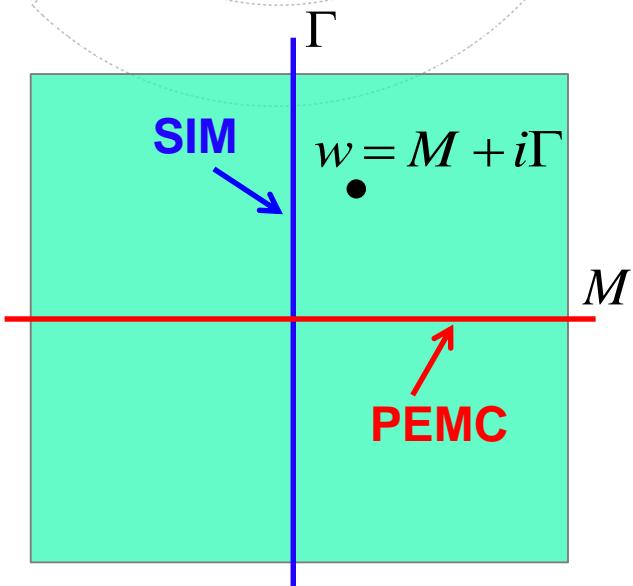


Geometrical representation of Tellegen media

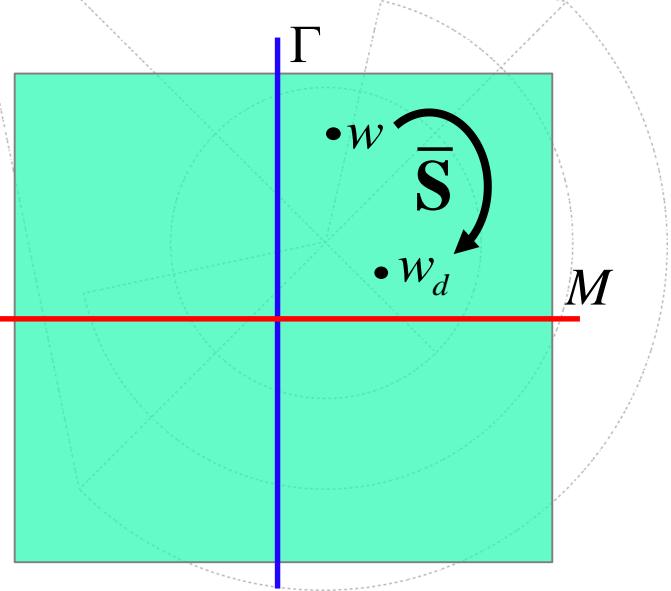
$$\begin{aligned}\mathbf{D} - M \mathbf{B} &= c^{-1} \Gamma |n| \mathbf{E} \\ \mathbf{H} + M \mathbf{E} &= c \Gamma |n| \mathbf{B}\end{aligned}$$

Families of isorefractive Tellegen media

$|n| = \text{const.}$



Geometrical interpretation of duality transformations



Duality transformations are Möbius mappings

$$w_d = (s_{22}w - s_{21}) / (s_{11} - s_{12}w)$$

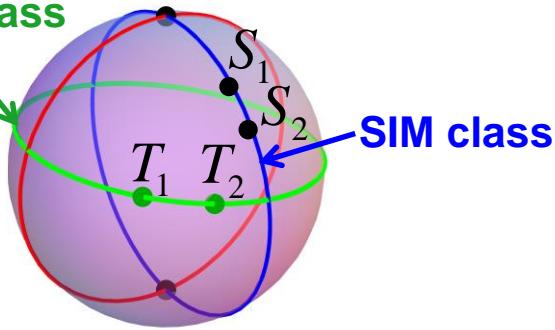
Reduction of two generic Tellegen media to simple media

Composition of two generalized rotations $\bar{\mathbf{S}}_{R_2} \cdot \bar{\mathbf{S}}_{R_1}$

$$T_1 \rightarrow (\varepsilon_1, \mu_1, \kappa_1)$$

$$T_2 \rightarrow (\varepsilon_2, \mu_2, \kappa_2)$$

Tellegen class

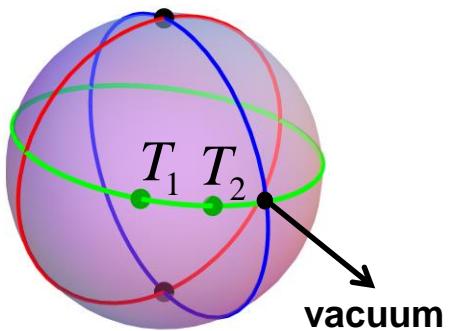


Reduction of two generic Tellegen media to simple media

Rotation angle

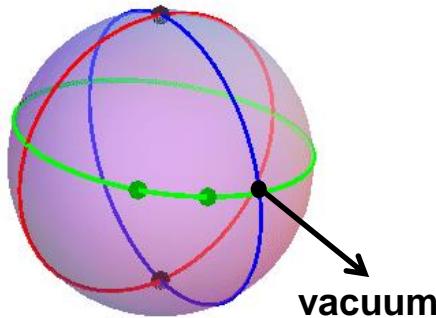
$$\theta_1 = \tan^{-1} \left[2\kappa_1 / (\mu_1 - \varepsilon_1) \right]$$

Rotation $\bar{\mathbf{S}}_{R_1} = e^{(\theta_1/2)\boldsymbol{\sigma} \cdot \hat{\mathbf{n}}}$



$$T_1 \rightarrow (\varepsilon_1, \mu_1, \kappa_1)$$

$$T_2 \rightarrow (\varepsilon_2, \mu_2, \kappa_2)$$

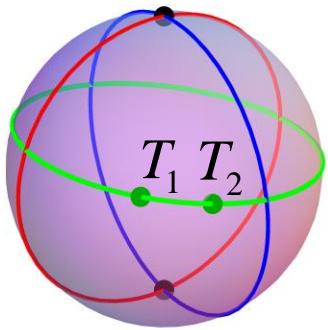


Reduction of two generic Tellegen media to simple media

Rotation angle

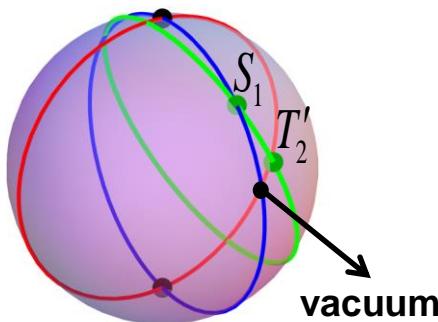
$$\theta_1 = \tan^{-1} \left[2\kappa_1 / (\mu_1 - \varepsilon_1) \right]$$

Rotation $\bar{\mathbf{S}}_{R_1} = e^{(\theta_1/2)\boldsymbol{\sigma} \cdot \hat{\mathbf{n}}}$



$$T_1 \rightarrow (\varepsilon_1, \mu_1, \kappa_1)$$

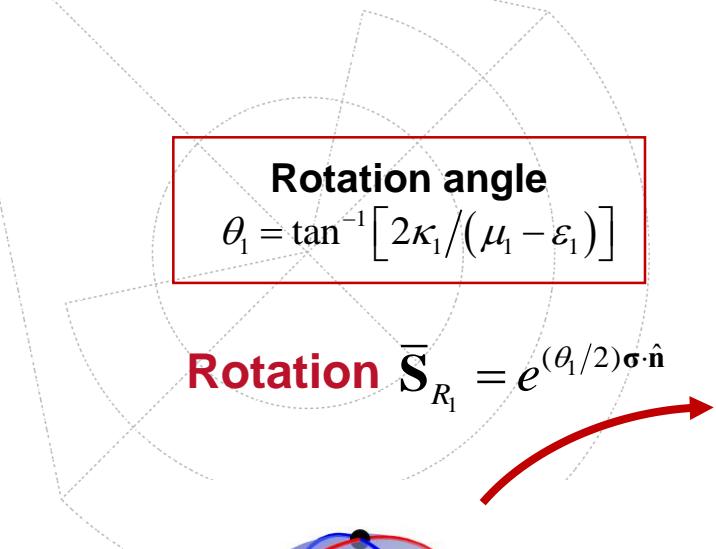
$$T_2 \rightarrow (\varepsilon_2, \mu_2, \kappa_2)$$



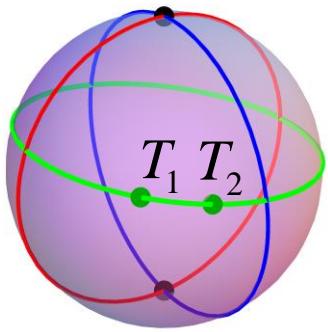
$$T_1 \rightarrow (\varepsilon'_1, \mu'_1)$$

$$T_2' \rightarrow (\varepsilon'_2, \mu'_2, \kappa'_2)$$

Reduction of two generic Tellegen media to simple media

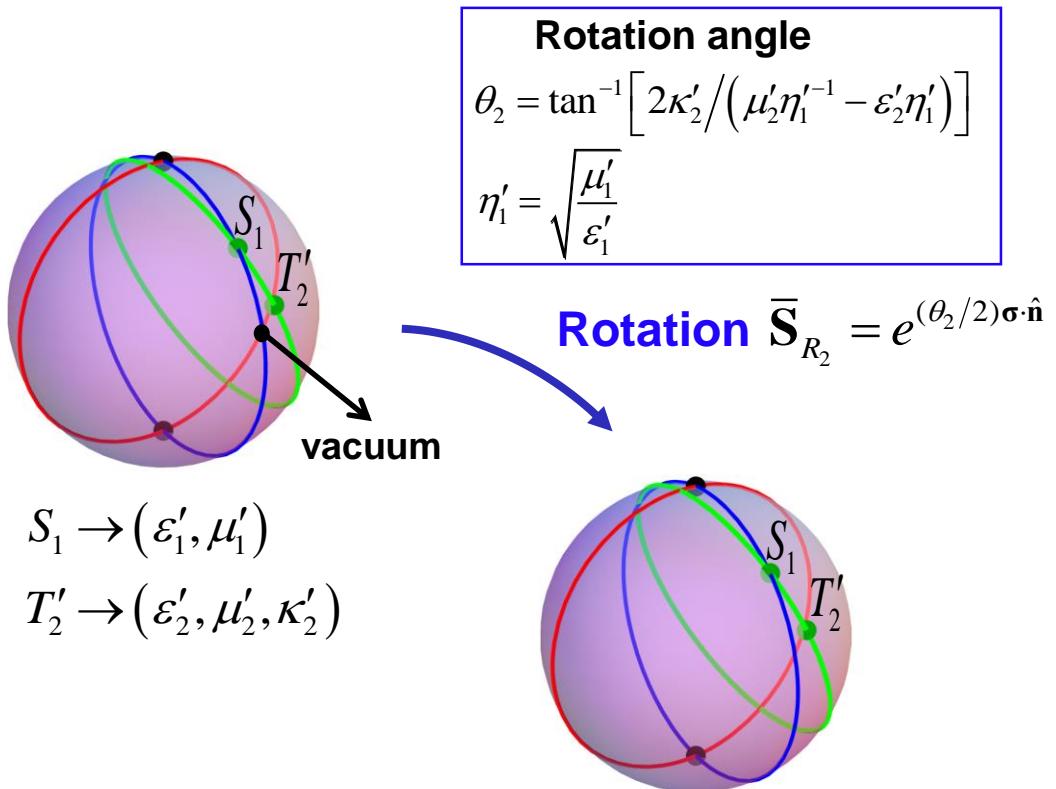


Rotation $\bar{\mathbf{S}}_{R_1} = e^{(\theta_1/2)\boldsymbol{\sigma} \cdot \hat{\mathbf{n}}}$

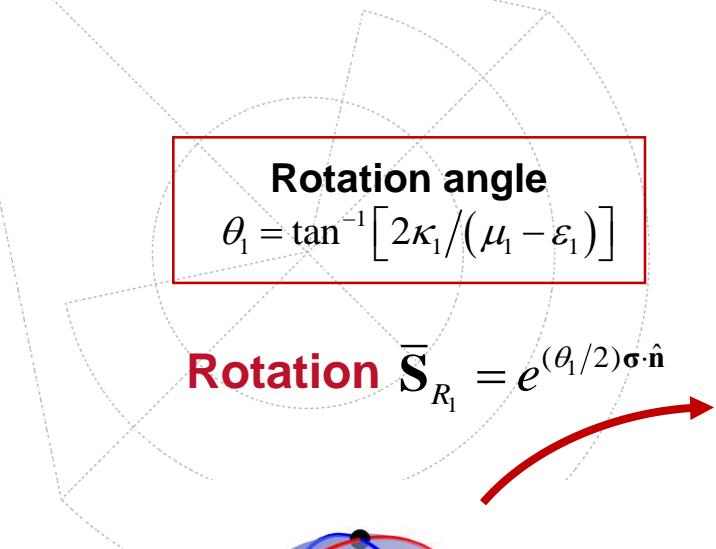


$T_1 \rightarrow (\varepsilon_1, \mu_1, \kappa_1)$

$T_2 \rightarrow (\varepsilon_2, \mu_2, \kappa_2)$

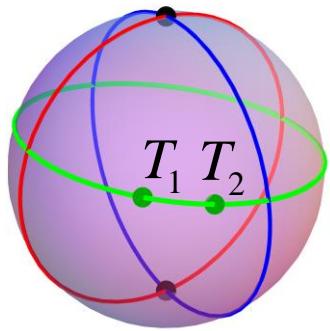


Reduction of two generic Tellegen media to simple media



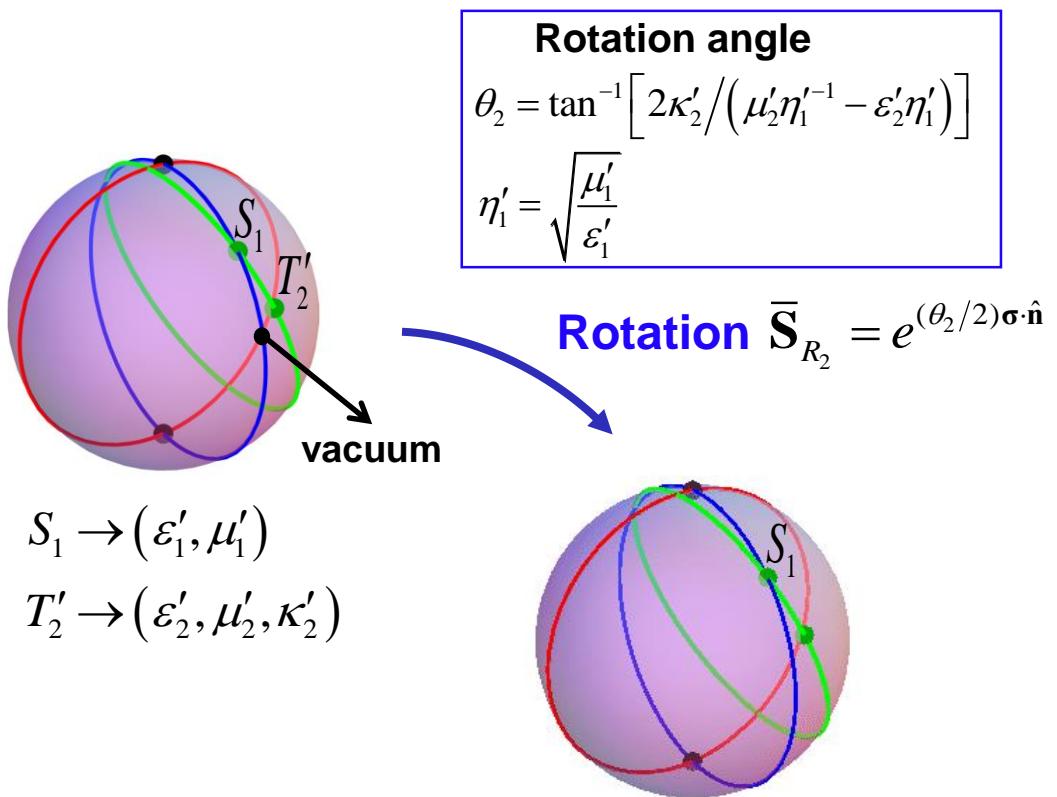
Rotation $\bar{\mathbf{S}}_{R_1} = e^{(\theta_1/2)\boldsymbol{\sigma} \cdot \hat{\mathbf{n}}}$

$$\text{Rotation angle} \\ \theta_1 = \tan^{-1} \left[2\kappa_1 / (\mu_1 - \varepsilon_1) \right]$$



$$T_1 \rightarrow (\varepsilon_1, \mu_1, \kappa_1)$$

$$T_2 \rightarrow (\varepsilon_2, \mu_2, \kappa_2)$$



$$S_1 \rightarrow (\varepsilon'_1, \mu'_1)$$

$$T'_2 \rightarrow (\varepsilon'_2, \mu'_2, \kappa'_2)$$

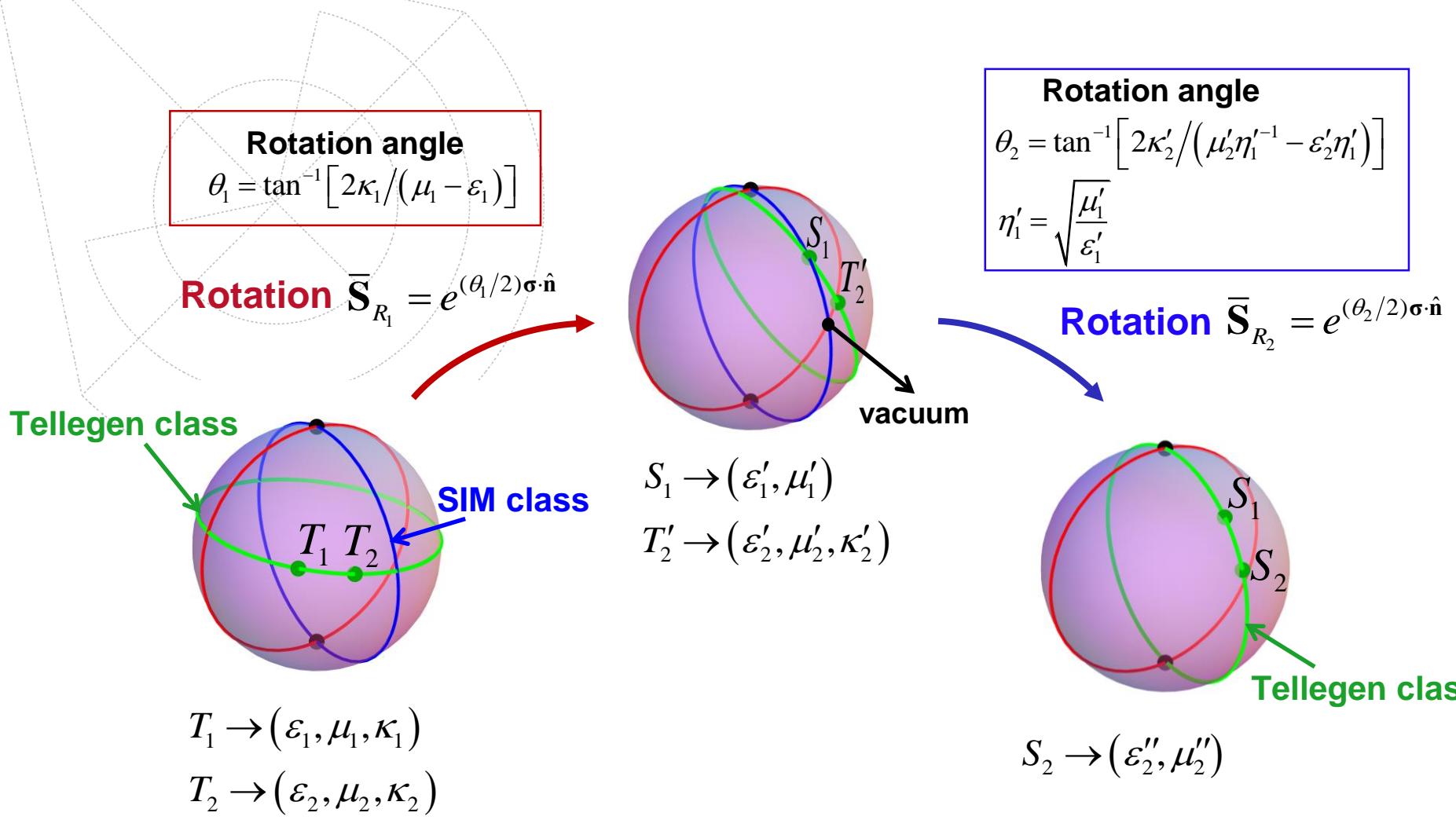
Rotation angle

$$\theta_2 = \tan^{-1} \left[2\kappa'_2 / (\mu'_2 \eta'^{-1} - \varepsilon'_2 \eta'_1) \right]$$

$$\eta'_1 = \sqrt{\frac{\mu'_1}{\varepsilon'_1}}$$

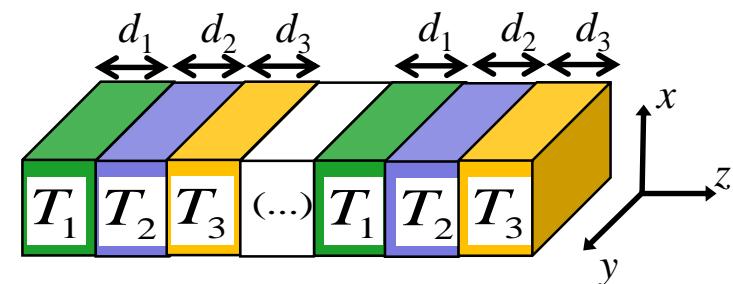
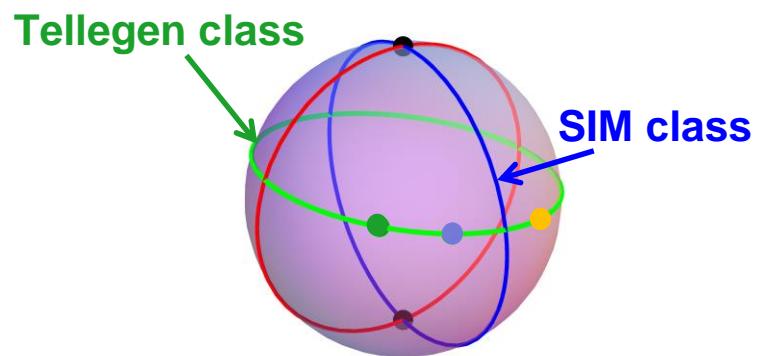
Rotation $\bar{\mathbf{S}}_{R_2} = e^{(\theta_2/2)\boldsymbol{\sigma} \cdot \hat{\mathbf{n}}}$

Reduction of two generic Tellegen media to simple media



Filipa R. Prudêncio, Sérgio A. Matos and Carlos R. Paiva, "Geometrical approach to duality transformations for Tellegen media," *IEEE Trans. Microwave Theory and Techn.*, vol. 62, no. 7, pp. 1417-1428, 2014.

Applications of Tellegen classes to periodic structures



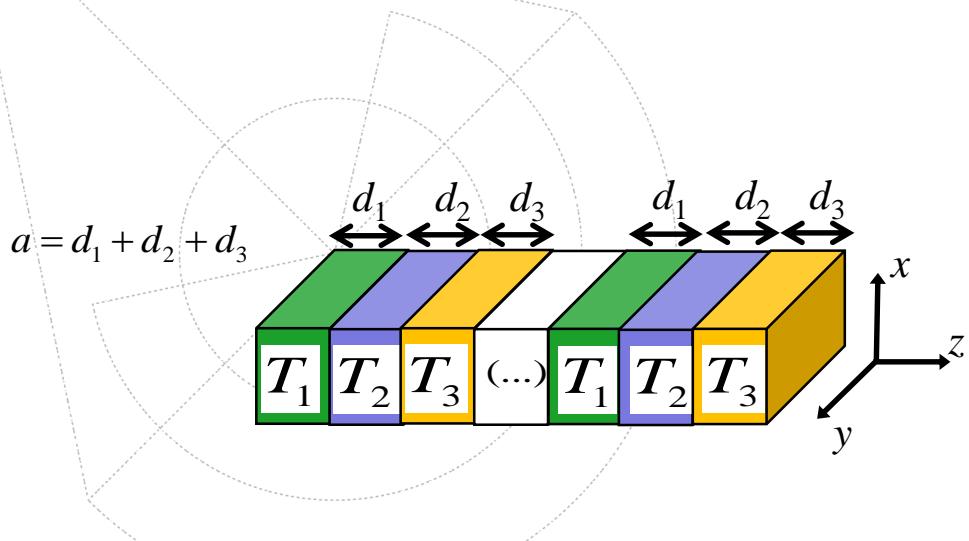
Dispersion equations and duality transformations

Duality transformations act over the material parameters and electromagnetic fields

Transformed material parameters $\bar{\mathbf{M}}_d = (\bar{\mathbf{S}}^{-1})^T \cdot \bar{\mathbf{M}} \cdot \bar{\mathbf{S}}^{-1}$

Transformed electromagnetic fields $\begin{pmatrix} \mathbf{E}_d \\ \eta_0 \mathbf{H}_d \end{pmatrix} = \bar{\mathbf{S}} \cdot \begin{pmatrix} \mathbf{E} \\ \eta_0 \mathbf{H} \end{pmatrix}$

Periodic structures with Tellegen media

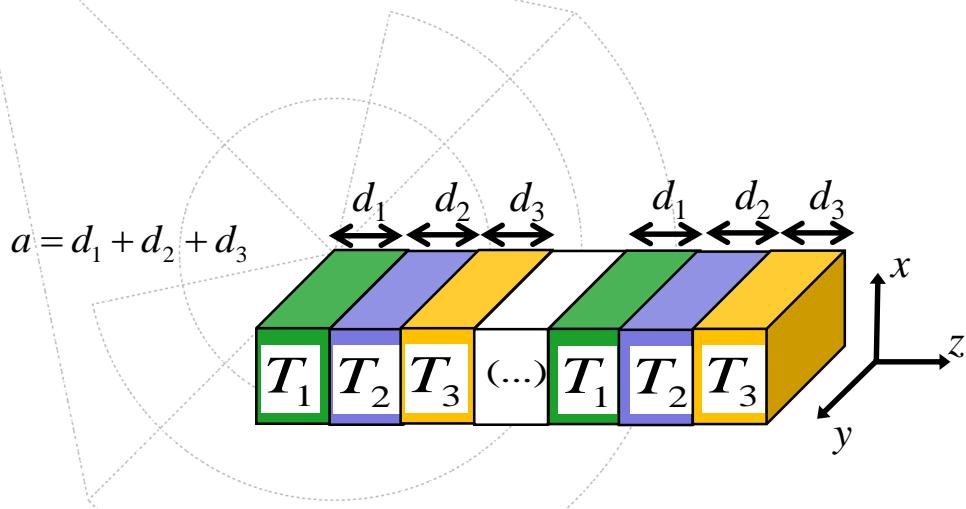


Dispersion equations

$$\cos(k_z a) = \frac{T_+}{2} + \frac{1}{2T_+} (1 - R_+^L R_+^R)$$

$$\cos(k_z a) = \frac{T_-}{2} + \frac{1}{2T_-} (1 - R_-^L R_-^R)$$

Periodic structures with Tellegen media



Dispersion equations

$$\cos(k_z a) = \frac{T_+}{2} + \frac{1}{2T_+} (1 - R_+^L R_+^R)$$

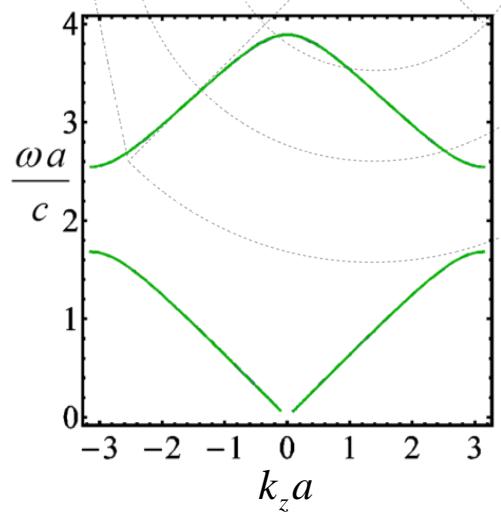
$$\cos(k_z a) = \frac{T_-}{2} + \frac{1}{2T_-} (1 - R_-^L R_-^R)$$

Periodic structures with Tellegen media

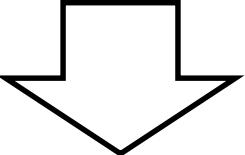
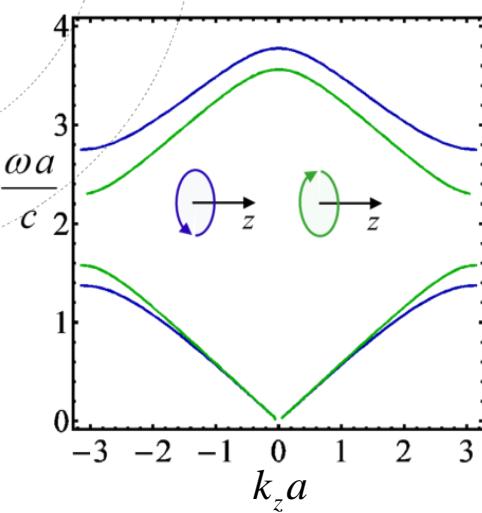
Symmetric band diagrams $\rightarrow \omega(k_z) = \omega(-k_z)$

Tellegen periodic structures

Same class

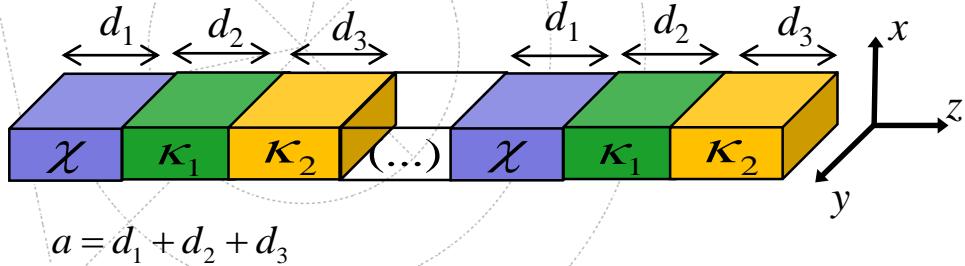


Different classes



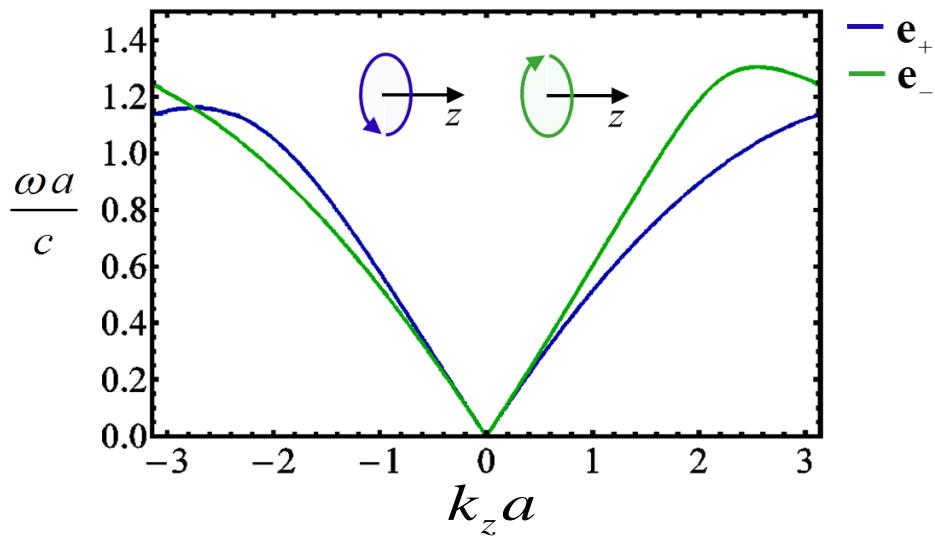
Periodic structures with bi-isotropic media

Periodic structures with chiral and Tellegen media



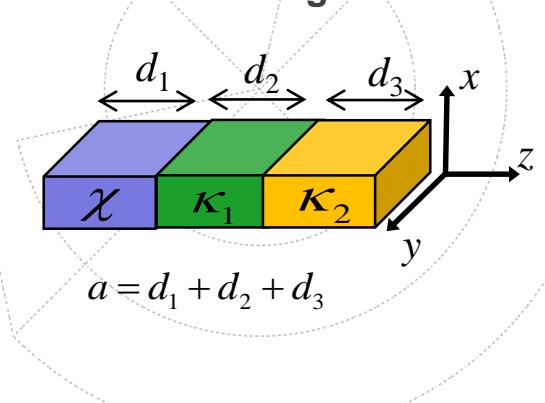
Asymmetric band diagrams

$$\omega(k_z) \neq \omega(-k_z)$$

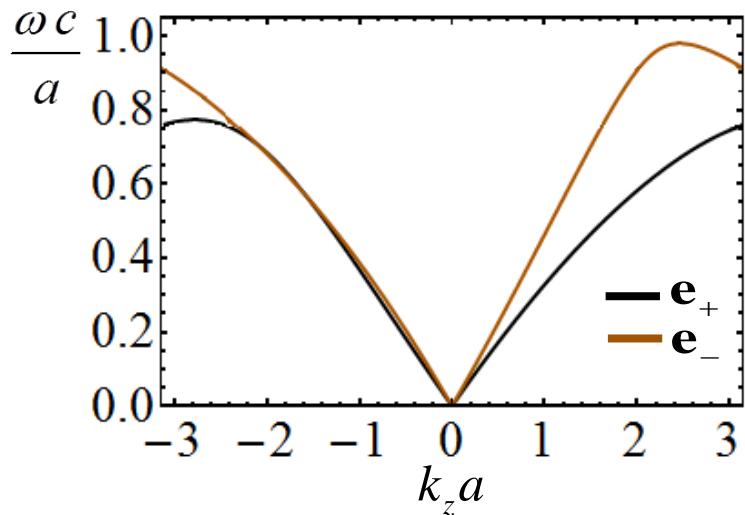
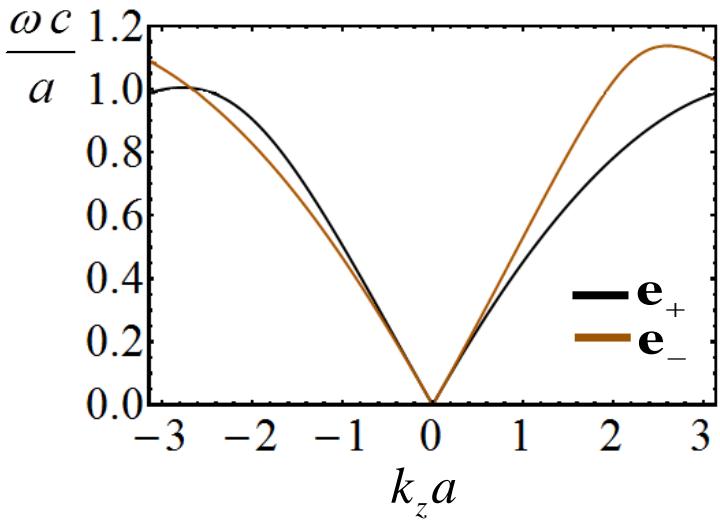
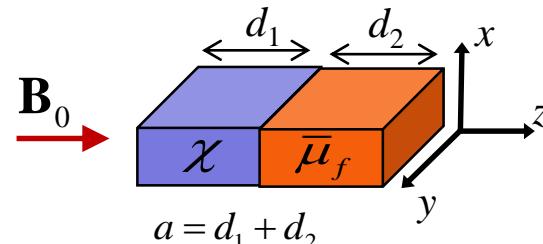


Periodic structures with nonreciprocal media

Periodic structures with chiral and Tellegen media



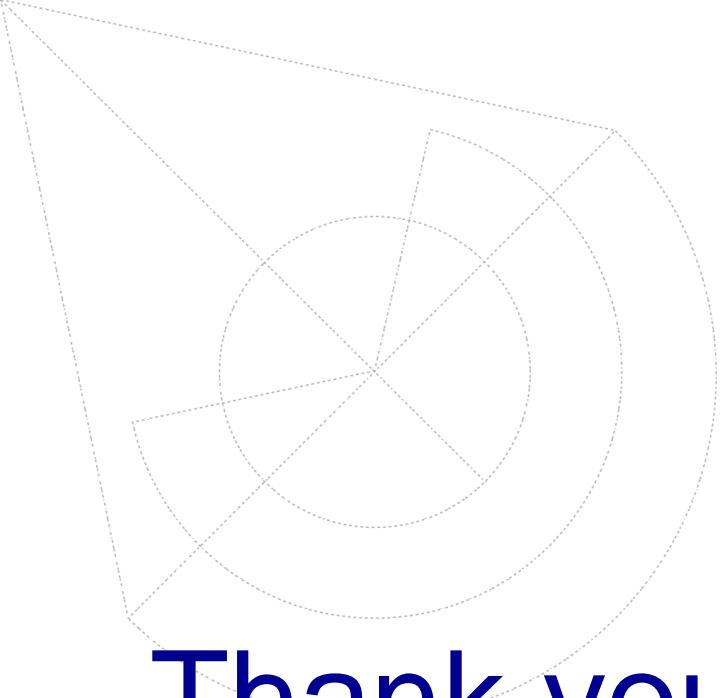
Periodic structures with chiral media and ferrites



Two Tellegen slabs can mimic one ferrite slab!

Conclusions

- Tellegen media are geometrically represented in the Riemann sphere
- Duality transformations are classified according to their geometric actions in the Riemann sphere
 - The most general classes of Tellegen media reducible to simple media were derived
 - Tellegen periodic structures not reducible to SIM periodic structures have nondegenerate Bloch modes
- Periodic structures with nonreciprocal and chiral media may have asymmetric band diagrams
 - Two Tellegen slabs can mimic one ferrite slab



Thank you for your attention!

Classification of duality transformations

Duality transformations

$$\rightarrow \bar{\mathbf{S}} = e^{u\boldsymbol{\sigma} \cdot \hat{\mathbf{n}}}$$

$$\boldsymbol{\sigma} \cdot \hat{\mathbf{n}} = \boldsymbol{\sigma}_x n_x + \boldsymbol{\sigma}_y n_y + \boldsymbol{\sigma}_z n_z$$

$$\hat{\mathbf{n}} = (n_x, n_y, n_z)$$

Pauli matrices

$$\boldsymbol{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \boldsymbol{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \boldsymbol{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\bar{\mathbf{S}} = e^{u\boldsymbol{\sigma} \cdot \hat{\mathbf{n}}} = \begin{cases} \cos u \bar{\mathbf{I}} + \sin u \boldsymbol{\sigma} \cdot \hat{\mathbf{n}}, & (\text{generalized rotations}) \quad \hat{\mathbf{n}} \cdot \hat{\mathbf{n}} = -1 \\ \cosh u \bar{\mathbf{I}} + \sinh u \boldsymbol{\sigma} \cdot \hat{\mathbf{n}}, & (\text{Lorentz boosts}) \quad \hat{\mathbf{n}} \cdot \hat{\mathbf{n}} = 1 \\ \bar{\mathbf{I}} + u \boldsymbol{\sigma} \cdot \hat{\mathbf{n}}, & (\text{Galilean boosts}) \quad \hat{\mathbf{n}} \cdot \hat{\mathbf{n}} = 0 \end{cases}$$

Tellegen classes

$$\bar{\mathbf{M}} = \begin{pmatrix} \varepsilon & \kappa \\ \kappa & \mu \end{pmatrix}$$

$\bar{\mathbf{S}}$

$$\bar{\mathbf{M}}_d = (\bar{\mathbf{S}}^{-1})^T \cdot \bar{\mathbf{M}} \cdot \bar{\mathbf{S}}^{-1}$$

Transformation matrix

$$\bar{\mathbf{S}} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$$

$$\kappa_{d_1} = \varepsilon_1 a + \mu_1 b + \kappa_1 c = 0$$

$$\kappa_{d_2} = \varepsilon_2 a + \mu_2 b + \kappa_2 c = 0$$

$$\kappa_{d_3} = \varepsilon_3 a + \mu_3 b + \kappa_3 c = 0$$

$$\rightarrow \begin{pmatrix} \varepsilon_1 & \mu_1 & \kappa_1 \\ \varepsilon_2 & \mu_2 & \kappa_2 \\ \varepsilon_3 & \mu_3 & \kappa_3 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0$$

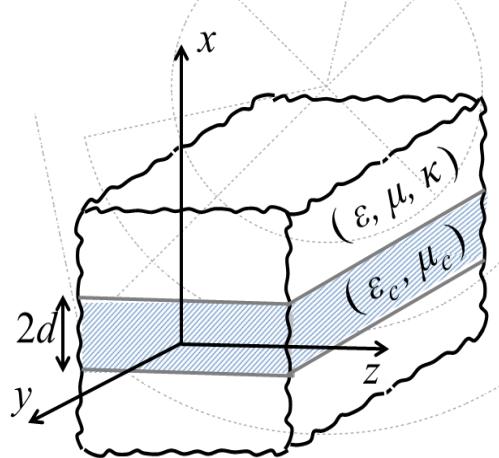
Classes of media

$$\det \begin{pmatrix} \varepsilon_1 & \mu_1 & \kappa_1 \\ \varepsilon_2 & \mu_2 & \kappa_2 \\ \varepsilon_3 & \mu_3 & \kappa_3 \end{pmatrix} = 0$$

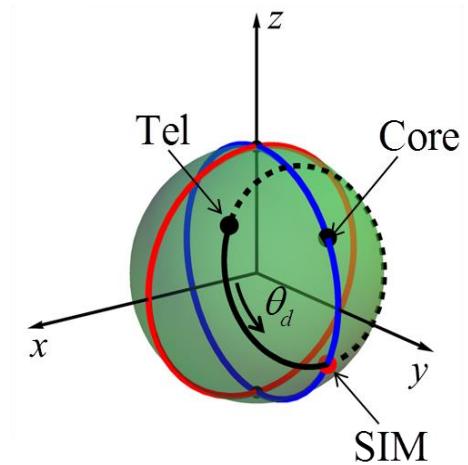
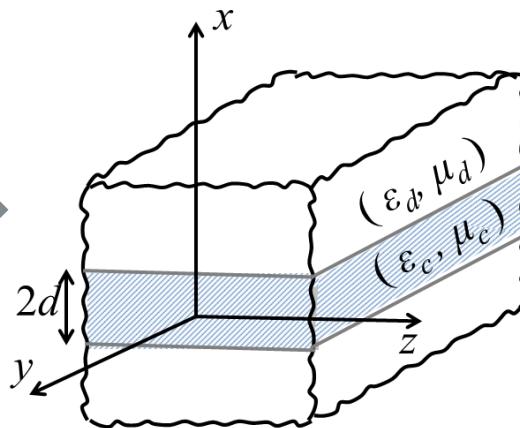
Example

Generalized rotation

$$\bar{\mathbf{S}}_R = e^{(\theta_d/2)\boldsymbol{\sigma} \cdot \hat{\mathbf{n}}}$$



$$\bar{\mathbf{S}}_R$$



Hybrid modes

$$\bar{\mathbf{S}}_R^{-1}$$

TE and TM modes

$$h^2 \sin^2(hd)n^2 + \alpha^2 \cos^2(hd)n_c^2 - h\alpha \cos(hd)\sin(hd)(\epsilon \mu_c + \epsilon_c \mu) = 0$$

$$\bar{\mathbf{S}}_R^{-1}$$

$$h^2 \sin^2(hd)n^2 + \alpha^2 \cos^2(hd)n_c^2 - h\alpha \cos(hd)\sin(hd)(\epsilon_d \mu_c + \epsilon_c \mu_d) = 0$$

Dispersion diagrams are invariant

Research on Tellegen media

- [1] I. E. Dzyaloshinskii, “On the magneto-electrical effect in antiferromagnetics,” *Sov Phys JETP*, vol. 10, pp. 628-629, 1960.
- [2] D. N. Astrov, “Magnetoelectric effect in chromium oxide,” *Sov Phys JETP*, vol. 13, no.4, pp. 729-733, 1961.
- [3] S. Coh and D. Vanderbilt, “B. B. Krichevtssov, V. V. Pavlov and V. N. Gridnev, “Spontaneous non-reciprocal reflection of light from antiferromagnetic Cr_2O_3 ,” *J. Phys.: Condens. Matter*, vol. 5, pp. 8233-8244, 1993.
- [4] A. Gosh, N. K. Sheridan and P. Fischer, “Janus particles with coupled electric and magnetic moments make a disordered magneto-electric medium,” arXiv:0708.1126, 2007.
- [5] S. Coh and D. Vanderbilt, “Canonical magnetic insulators with isotropic magnetoelectric coupling,” *Phys. Rev. B*, vol. 88, pp. 121106, 2013.
- [6] X. L. Qi, R. Li, J. Zang and S. C. Zhang, “Inducing a magnetic monopole with topological surface states,” *Science*, vol. 323, pp. 1184-1187, 2009.
- [7] X. L. Qi and S. C. Zhang, “Topological insulators and superconductors,” *Rev. Mod. Phys.*, vol. 83, pp. 1057-1110, 2011.
- [8] F. Liu, J. Xu and Y. Yang, “Polarization conversion of reflected electromagnetic wave from topological insulator,” *J. Opt. Soc. Am. B*, vol. 31, no. 4, pp. 735-741, 2014.

Research on duality transformations

- [1] I. V. Lindell, *Methods for Electromagnetic Field Analysis*. Oxford, Clarendon Press, 1992.
- [2] A. N. Serdyukov, I. V. Semchenko, S. A. Tretyakov and A. H. Sihvola, *Electromagnetics of Bi-anisotropic Materials: Theory and Applications*. Amsterdam, Gordon and Breach Science Publishers, 2001.
- [3] I. V. Lindell, A. H. Sihvola, S. A. Tretyakov and A. J. Viitanen, *Electromagnetic Waves in Chiral and Bi-Isotropic Media*. Boston, MA: Artech House, 1994.
- [4] A. N. Serdyukov, A. H. Sihvola, S. A. Tretyakov and I. V. Semchenko, “Duality in Electromagnetics: Application to Tellegen Media,” *Electromagnetics*, vol. 16, no. 1, pp. 51-61, 1996.
- [5] A. H. Sihvola, S. A. Tretyakov, A. N. Serdyukov and I. V. Semchenko, “Duality Once More Applied to Tellegen Media,” *Electromagnetics*, vol. 17, no. 2, pp. 205-211, 1997.
- [6] A. Lakhtakia and W. S. Weiglhofer, “On the Application of Duality to Tellegen Media,” *Electromagnetics*, vol. 17, no. 2, pp. 199-204, 1997.
- [7] I. V. Lindell and A. H. Sihvola, “Transformation method for problems involving perfect electromagnetic conductor (PEMC) structures,” *IEEE Trans. Antennas Propag.*, vol. 53, no. 9, pp. 3005-3011, Sep. 2005.

Research on photonic crystals

- [1] A. Figotin and I. Vitebsky, "Nonreciprocal magnetic photonic crystals," *Phys. Rev. E*, vol. 63, pp. 066609, 2001.
- [2] A. Figotin and I. Vitebsky, "Frozen light in photonic crystals with degenerate band edge," *Phys. Rev. E*, vol. 74, pp. 066613, 2006.
- [3] V. R. Tuz and S. L. Prosvirnin, "Polarization switching and nonreciprocity in symmetric and asymmetric magnetophotonic multilayers with nonlinear defect," *Phys. Rev. A*, vol. 63, pp. 043822, 2012.