



MATERIAIS INTELIGENTES  
PARA A RADIOCIÊNCIA

17º

CONGRESSO DO COMITÉ  
PORTUGUÊS DA URSI

## Electromagnetic Solution of Large and Deep-multiscale Problems using Surface Integral Equation Techniques

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<sup>1</sup>Dept. Tecnología de Computadores y Comunicaciones, **University of Extremadura**, Spain

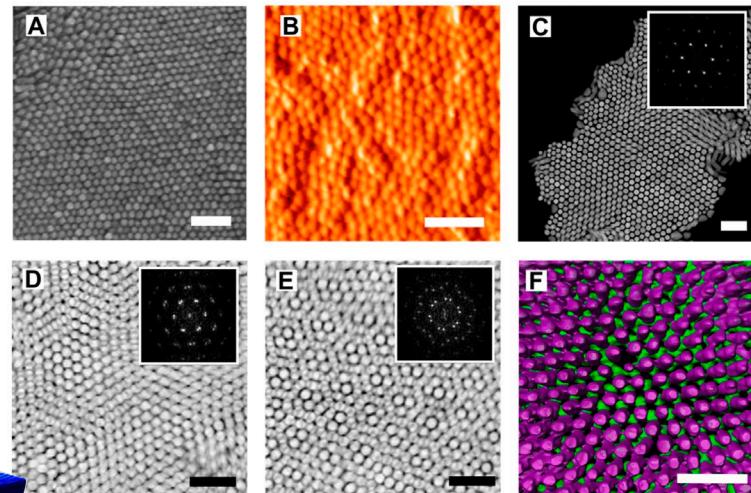
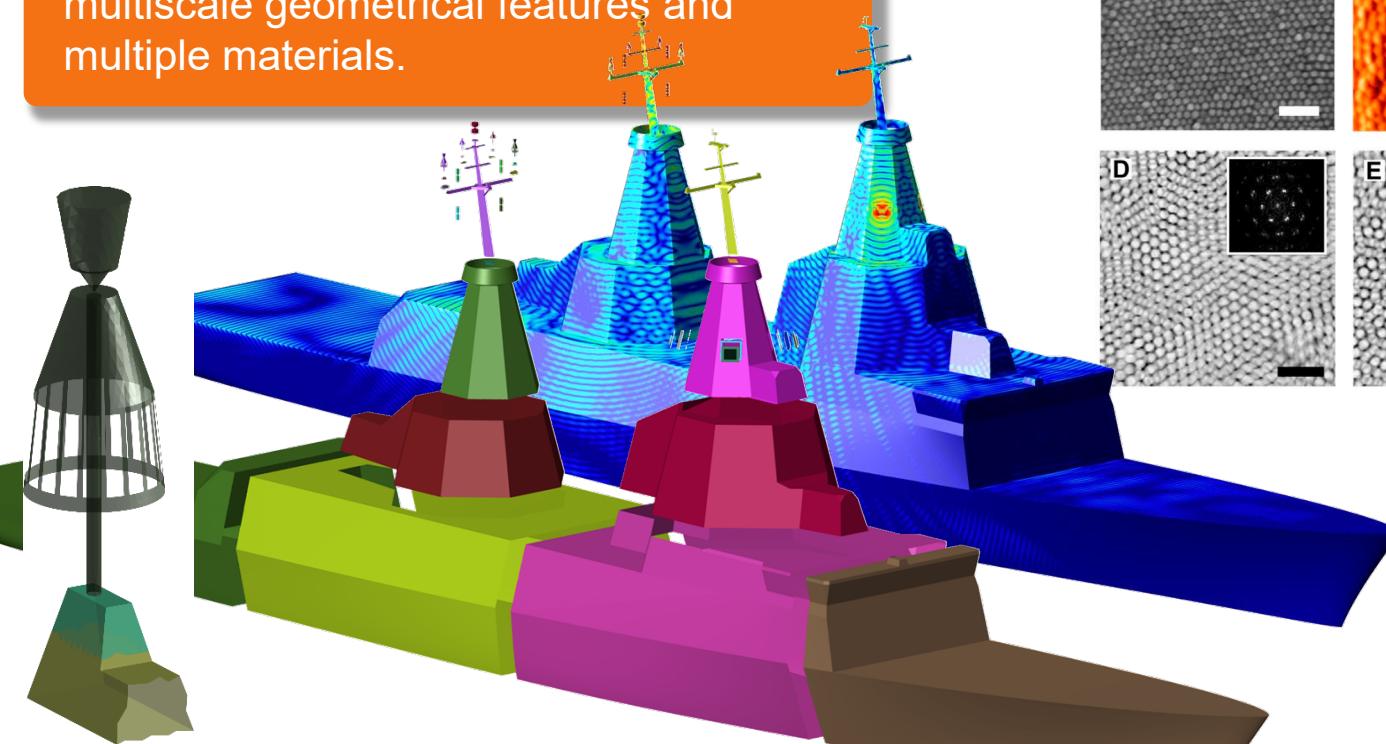
<sup>2</sup>Dept. Teoría de la Señal y Comunicaciones, **University of Vigo**, Spain

# Where we are...



# Motivation and objectives

Solving real-life problems involving complicated assemblies, including multiscale geometrical features and multiple materials.



# Fast & accurate solution of large multiscale problems

Surface integral equations (SIE)  
Method of Moments

Multilevel fast multipole algorithm – fast  
Fourier transform (MLFMA – FFT)

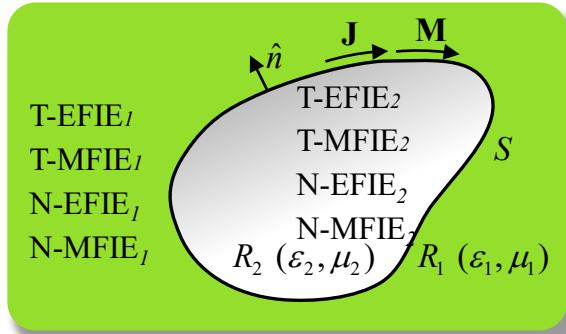
Highly scalable parallelization

Domain decomposition method (DDM)

Discontinuous Galerkin (DG)

Multiresolution quasi-Helmholtz  
decomposition

# Surface Integral Equations (SIE) based Method of Moments



$$\left. \begin{aligned} \text{T-EFIE}_l : & \left( \eta_l \mathcal{L}_l(\mathbf{J}_l) - \mathcal{K}_l(\mathbf{M}_l) \right)_{\tan} + \frac{1}{2} \hat{\mathbf{n}}_l \times \mathbf{M}_l = \left( -\mathbf{E}_l^{\text{inc}} \right)_{\tan} \\ \text{T-MFIE}_l : & \left( \mathcal{K}_l(\mathbf{J}_l) + \frac{1}{\eta_l} \mathcal{L}_l(\mathbf{M}_l) \right)_{\tan} - \frac{1}{2} \hat{\mathbf{n}}_l \times \mathbf{J}_l = \left( -\mathbf{H}_l^{\text{inc}} \right)_{\tan} \\ \text{N-EFIE}_l : & \hat{\mathbf{n}}_l \times \left( \eta_l \mathcal{L}_l(\mathbf{J}_l) - \mathcal{K}_l(\mathbf{M}_l) \right) - \frac{1}{2} \mathbf{M}_l = \hat{\mathbf{n}}_l \times \mathbf{E}_l^{\text{inc}} \\ \text{N-MFIE}_l : & \hat{\mathbf{n}}_l \times \left( \mathcal{K}_l(\mathbf{J}_l) + \frac{1}{\eta_l} \mathcal{L}_l(\mathbf{M}_l) \right) + \frac{1}{2} \mathbf{J}_l = \hat{\mathbf{n}}_l \times \mathbf{H}_l^{\text{inc}} \end{aligned} \right\}$$

$-\hat{\mathbf{n}} \times \hat{\mathbf{n}} \times \rightarrow$  Tangential (T)-formulations

$\hat{\mathbf{n}} \times \rightarrow$  Twisted or normal (N)-formulations

Stratton-Chu integro-differential operators

$$\mathcal{L}(\mathbf{X}) = jk \int_S \mathbf{X}(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') d\mathbf{s}' - \frac{1}{jk} \nabla \int_S \nabla' \cdot \mathbf{X}(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') d\mathbf{s}'$$

$$\mathcal{K}(\mathbf{X}) = PV \int_S \mathbf{X}(\mathbf{r}') \times \nabla G(\mathbf{r}, \mathbf{r}') d\mathbf{s}'$$

Green's function

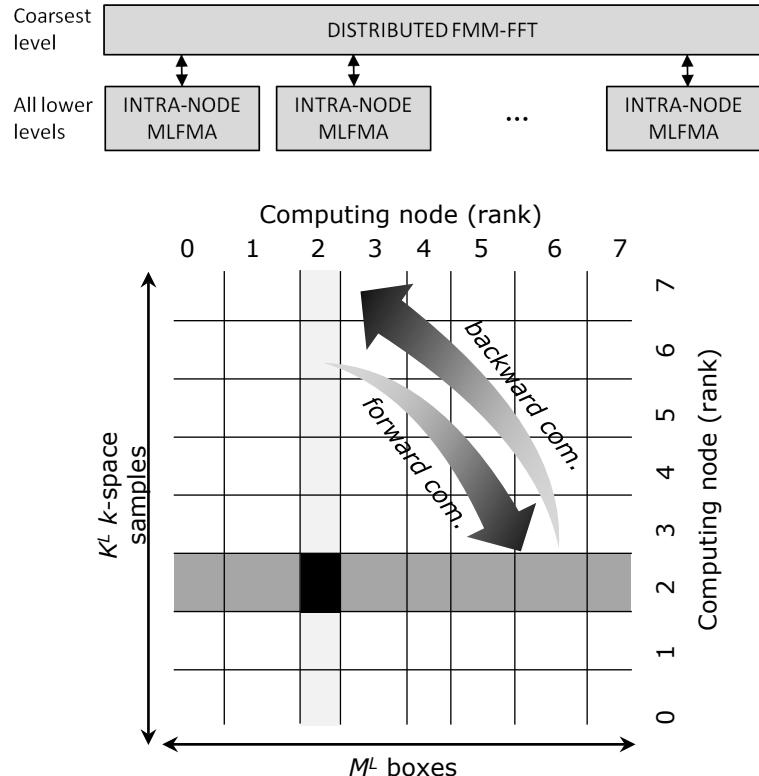
$$G(\mathbf{r}, \mathbf{r}') = \frac{e^{-jkr-r'}}{4\pi |\mathbf{r} - \mathbf{r}'|}$$

$$\text{JCFIE: } \sum_{l=1}^2 a_l \frac{1}{\eta_l} \text{T-EFIE}_l + \sum_{l=1}^2 b_l \text{N-MFIE}_l$$

$$\text{MCFIE: } \sum_{l=1}^2 c_l \text{N-EFIE}_l + \sum_{l=1}^2 d_l \eta_l \text{T-MFIE}_l$$

Galerkin  
Method of  
moments

# Parallel-prone MLFMA-FFT



## MVP

MLFMA: aggregation, interpolation, near-field translation

Forward communication

FMM-FFT: 3D-FFT far-field translation

Backward communication

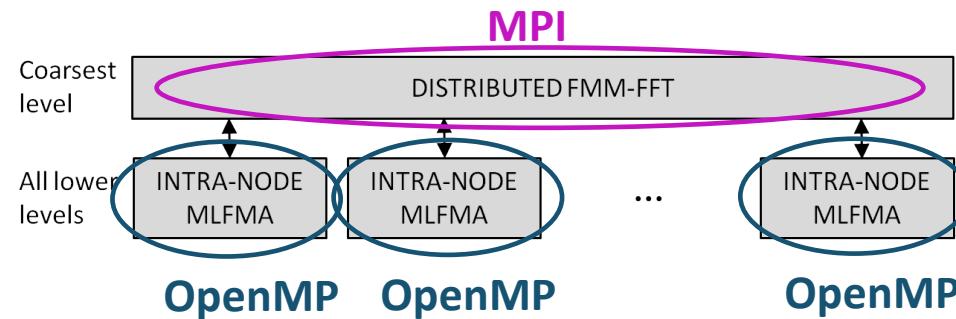
MLFMA: anterpolation, disaggregation

Communication of noncontiguous data layout, because of the distributed transposition.

Nonuniform communication volumes, because the size of the actual transferred data to each node differs.

## Hybrid parallel programming

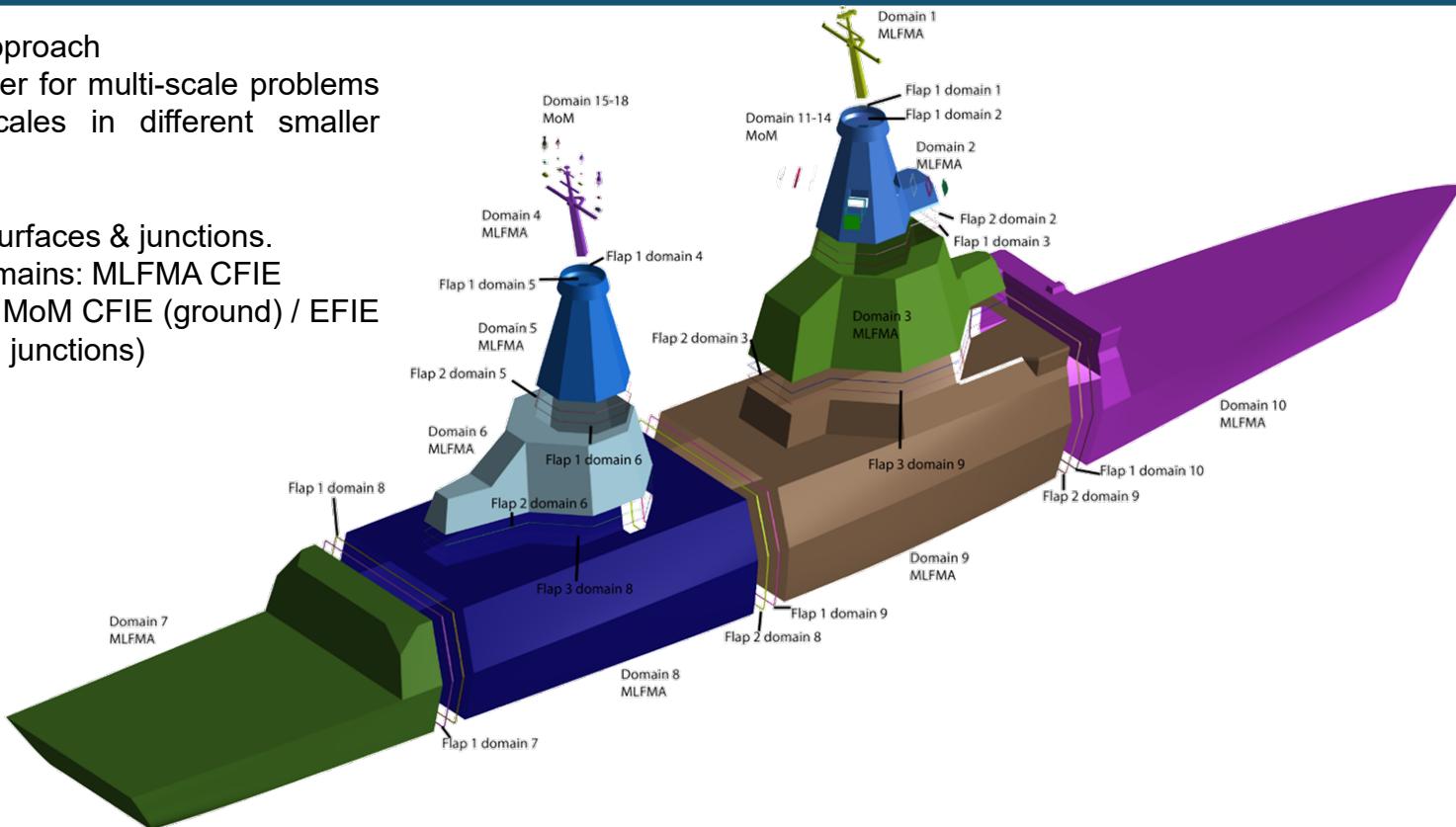
- Message passing interface (MPI) for distributed computations (FMM-FFT)
- OpenMP standard for shared-memory computations (MLFMA)
- Optimal scheme for mixed-memory architectures



J. M. Taboada, M. G. Araújo, F. Obelleiro, J. L. Rodríguez, L. Landesa, "MLFMA-FFT parallel algorithm for the solution of extremely large problems in electromagnetics," *Proceedings of the IEEE*, vol. 101, no. 2, pp. 350-363, Feb. 2013.

# Domain Decomposition Method (DDM)

- Divide and conquer approach
- Excellent preconditioner for multi-scale problems (isolating different scales in different smaller subdomains)
- 14 subdomains, 121 surfaces & junctions.
- Superstructure subdomains: MLFMA CFIE
- Antenna subdomains: MoM CFIE (ground) / EFIE (patch & patch-ground junctions)



D. M. Solís, V. F. Martín, M. G. Araújo, D. Larios, F. Obelleiro, and J. M. Taboada, "Accurate EMC Engineering on Realistic Platforms Using an Integral Equation Domain Decomposition Approach," *IEEE Trans. Antennas Propag.*, vol. 68, no.4, pp. 3002 – 3015, Apr. 2020,

# Domain Decomposition (DDM)

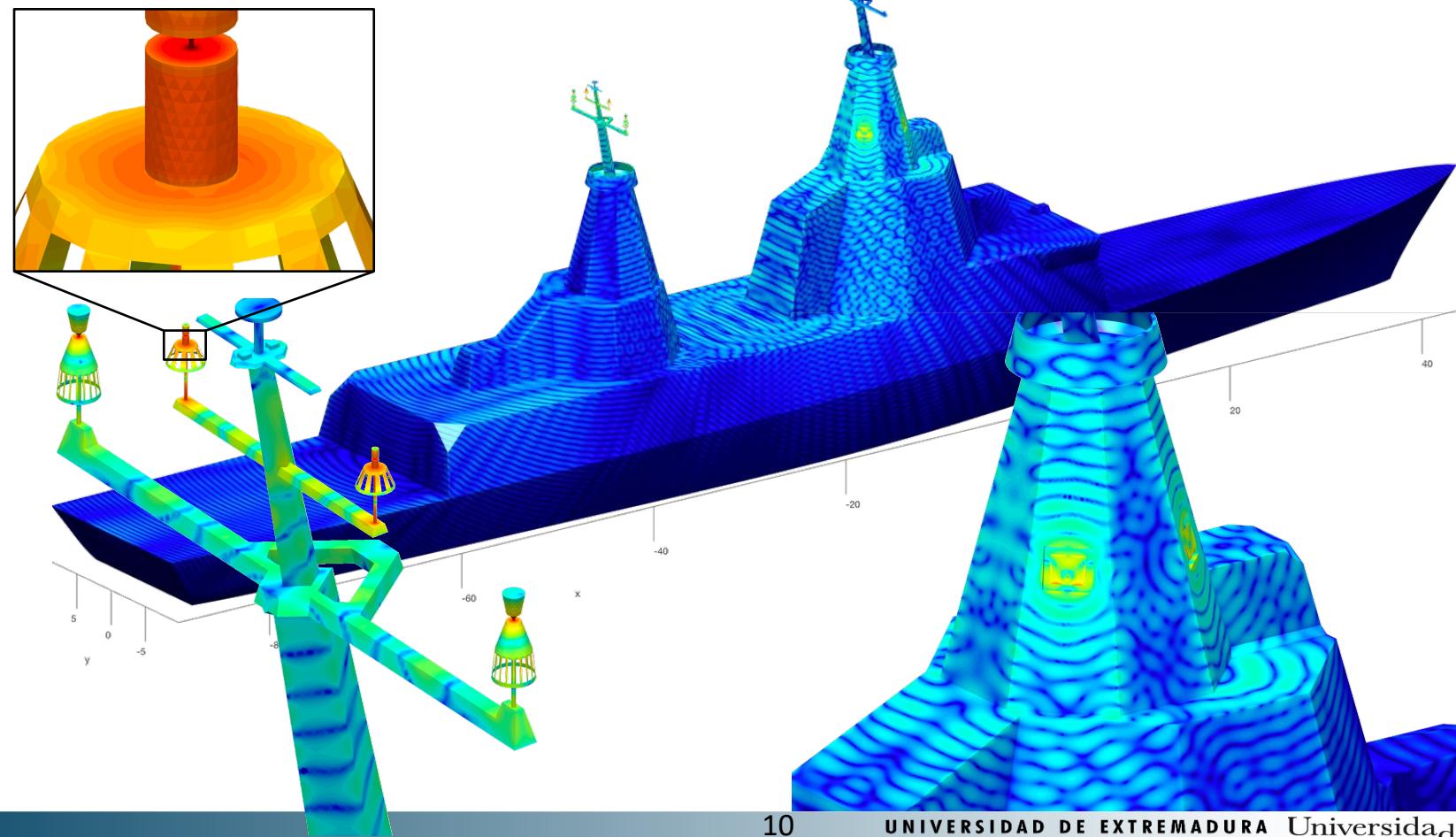
$$\mathbf{Z} \cdot \mathbf{I} = \mathbf{V}$$

$$\mathbf{P} \cdot \mathbf{Z} \cdot \mathbf{I} = \mathbf{P} \cdot \mathbf{V}$$

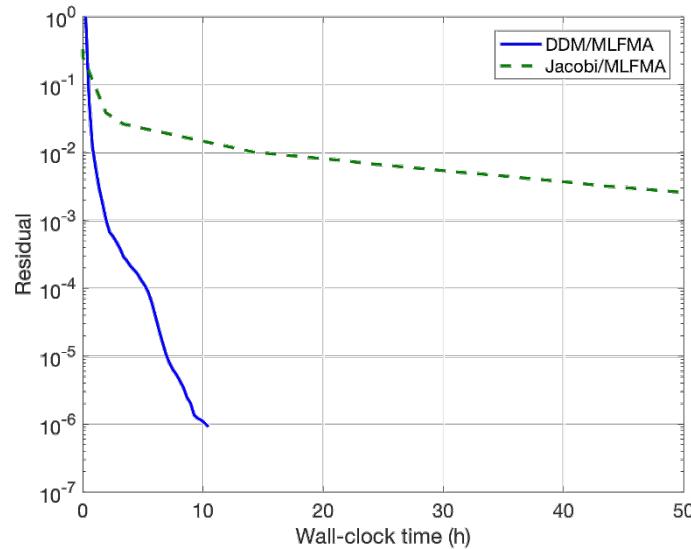
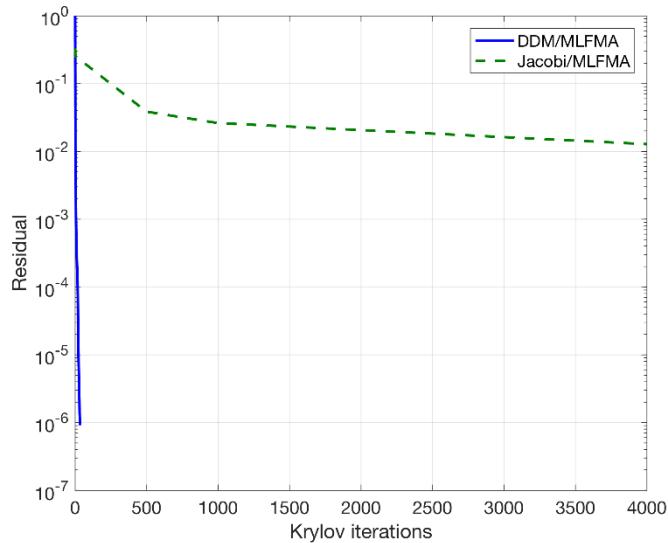
$$\mathbf{P} = \begin{bmatrix} \mathbf{Z}_1^{-1} & & & 0 \\ & \mathbf{Z}_2^{-1} & & \\ & & \ddots & \\ 0 & & & \mathbf{Z}_N^{-1} \end{bmatrix}$$

D. M. Solís, V. F. Martín, M. G. Araújo, D. Larios, F. Obelleiro, and J. M. Taboada, "Accurate EMC Engineering on Realistic Platforms Using an Integral Equation Domain Decomposition Approach," *IEEE Trans. Antennas Propag.*, vol. 68, no.4, pp. 3002 – 3015, Apr. 2020,

# Solving a multiscale radiation problem



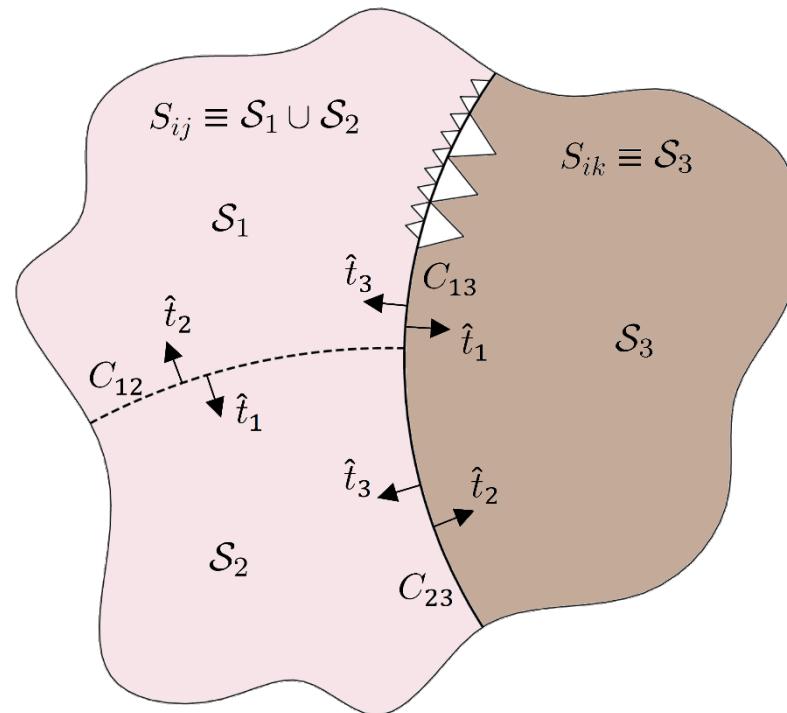
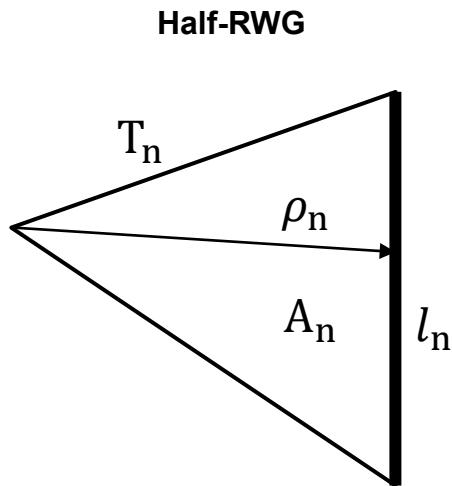
# Solving a multiscale radiation problem. Speeding-up convergence.



- 4 Intel Xeon E7-8867v3 microprocessors, with a total of 64 cores (no hyperthreading) and 1 TB RAM.
- Residual error up to  $10^{-6}$  in 10 hours
- DDM is not parallelized, but the MLFMA-FFT solver of the local sub-domains
- The **speed-up is only due to the improvement of convergence posed by DDM**, not due to the DDM parallel propensity.

# Discontinuous Galerking (DG)

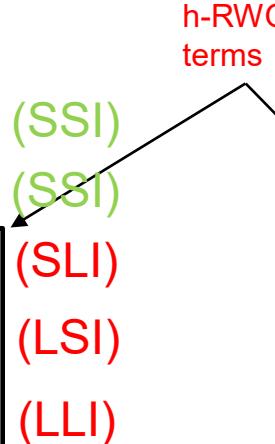
- We apply DG to the collection of surfaces composing the entire problem.
- Nonconformal meshes are allowed across the tear lines and junctions between different surfaces.



# Discontinuous Galerking (DG)

- Product rule for divergence
- Divergence Gauss theorem
- half-RWG normal component does not vanish at the boundary contour (non-div-conforming basis/testing function)

$$A_{mn}^i = \int_{\Delta_m} \mathbf{f}_m \cdot \mathcal{L}_i(\mathbf{f}_n) dS = \\ \int_{\Delta_m} \mathbf{f}_m \int_{\Delta_n} \mathbf{f}_n g_i(\mathbf{r}, \mathbf{r}') dS' dS \\ - \frac{1}{jk_i} \left[ \int_{\Delta_m} \nabla \cdot \mathbf{f}_m \int_{\Delta_n} \nabla' \cdot \mathbf{f}_n g_i(\mathbf{r}, \mathbf{r}') dS' dS \right. \\ \left. - \int_{\Delta_m} \nabla \cdot \mathbf{f}_m \oint_{\partial S_n} \hat{\mathbf{m}}_n \cdot \mathbf{f}_n g_i(\mathbf{r}, \mathbf{r}') d\partial S' dS \right. \\ \left. - \oint_{\partial S_m} \hat{\mathbf{m}}_m \cdot \mathbf{f}_m \int_{\Delta_{n|i}} \nabla' \cdot \mathbf{f}_n g_i(\mathbf{r}, \mathbf{r}') dS' d\partial S \right. \\ \left. + \oint_{\partial S_m} \hat{\mathbf{m}}_m \cdot \mathbf{f}_m \oint_{\partial S_n} \hat{\mathbf{m}}_n \cdot \mathbf{f}_n g_i(\mathbf{r}, \mathbf{r}') d\partial S' d\partial S \right]$$



$$\mathcal{L}_i(\mathbf{X}_i) = jk_i \int_S \mathbf{X}_i(\mathbf{r}') g_i(\mathbf{r}, \mathbf{r}') dS' \\ + \frac{1}{jk_i} \nabla \int \mathbf{X}_i(\mathbf{r}') \nabla' g_i(\mathbf{r}, \mathbf{r}') dS'$$

$$A_{mn}^{ri} = \int_{\Delta_m} \mathbf{f}_m \cdot \hat{\mathbf{n}}_m \times \mathcal{L}_i(\mathbf{f}_n) dS = \\ \int_{\Delta_m} (\mathbf{f}_m \times \hat{\mathbf{n}}_m) \int_{\Delta_n} \mathbf{f}_n g_i(\mathbf{r}, \mathbf{r}') dS' dS \\ - \frac{1}{jk_i} \left[ \int_{\Delta_m} \nabla \cdot (\mathbf{f}_m \times \hat{\mathbf{n}}_m) \int_{\Delta_n} \nabla' \cdot \mathbf{f}_n g_i(\mathbf{r}, \mathbf{r}') dS' dS \right. \\ \left. - \int_{\Delta_m} \nabla \cdot (\mathbf{f}_m \times \hat{\mathbf{n}}_m) \oint_{\partial S_n} \hat{\mathbf{m}}_n \cdot \mathbf{f}_n g_i(\mathbf{r}, \mathbf{r}') d\partial S' dS \right] \\ - \oint_{\partial S_m} \hat{\mathbf{m}}_m \cdot (\mathbf{f}_m \times \hat{\mathbf{n}}_m) \int_{\Delta_n} \nabla' \cdot \mathbf{f}_n g_i(\mathbf{r}, \mathbf{r}') dS' d\partial S \\ \left. + \oint_{\partial S_m} \hat{\mathbf{m}}_m \cdot (\mathbf{f}_m \times \hat{\mathbf{n}}_m) \oint_{\partial S_n} \hat{\mathbf{m}}_n \cdot \mathbf{f}_n g_i(\mathbf{r}, \mathbf{r}') d\partial S' d\partial S \right]$$

(SSI)

(SSI)

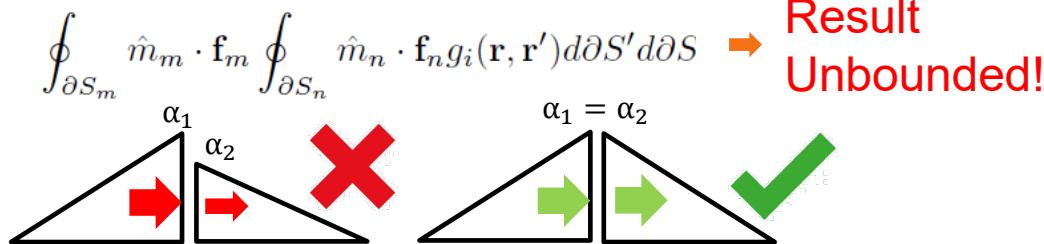
(SLI)

(LSI)

(LLI)

[1] V. F. Martín, L. Landesa, F. Obelleiro, and J. M. Taboada, "A Discontinuous Galerkin Combined Field Integral Equation Formulation for Electromagnetic Modeling of Piecewise Homogeneous Objects of Arbitrary Shape," *IEEE Trans. Antennas Propag.*, vol 70, no 1, pp. 487 - 498 Jan. 2022

# Interior Penalty (IP)

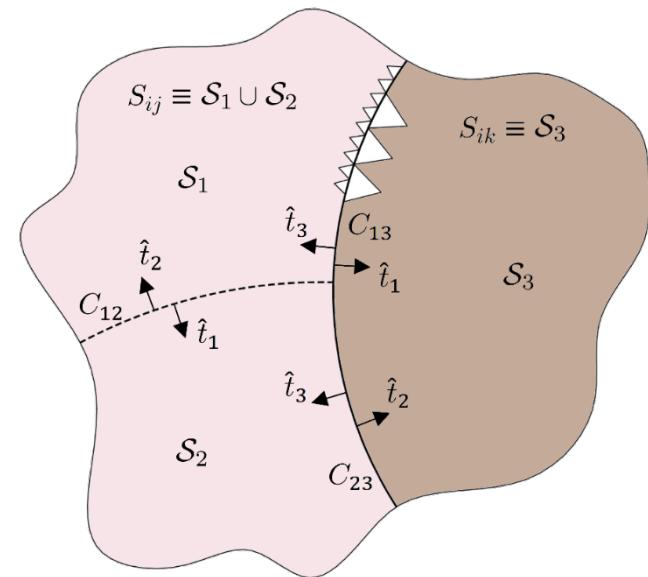


IP term: normal current continuity

$$\hat{t}_k \cdot \mathbf{X}_k + \hat{t}_{k'} \cdot \mathbf{X}_{k'} = 0 \quad \text{on } C_{kk'}$$

$$\sum_{\substack{\partial S_m \\ \in C_k}} \sum_{\substack{\partial S_n \\ \in C_{k'}}} (\hat{m}_m \cdot \mathbf{f}_m X_m + \hat{m}_n \cdot \mathbf{f}_n X_n) = 0 \quad \text{on } C_{kk'}$$

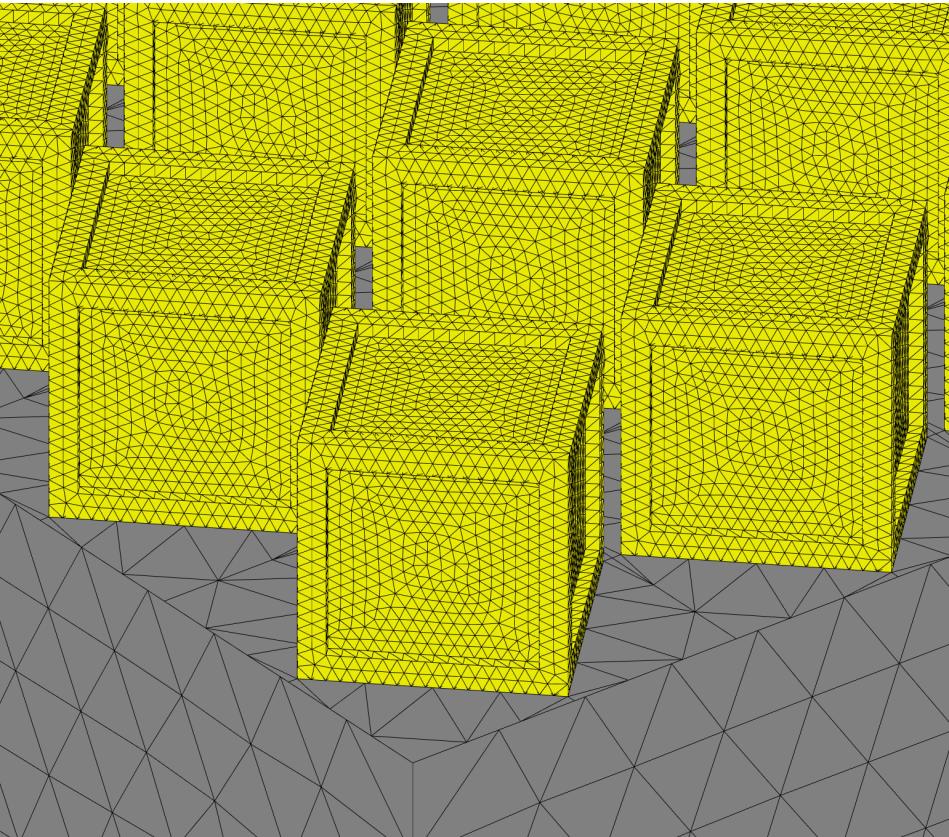
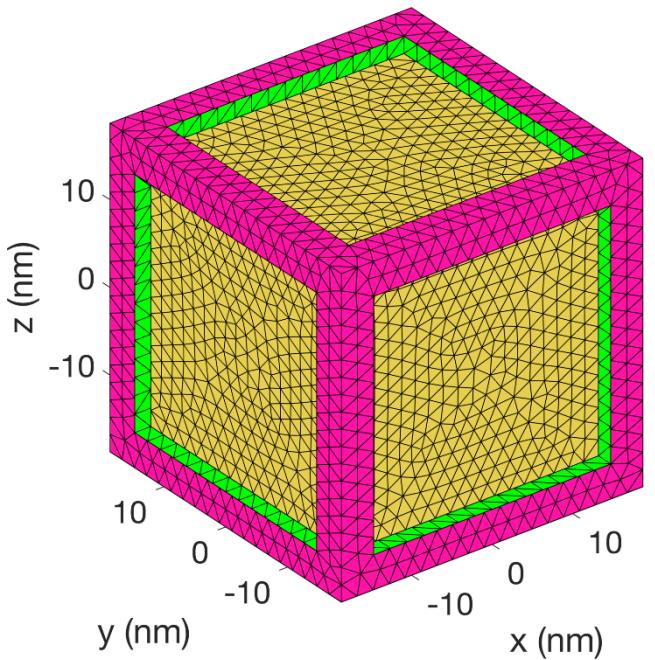
$$\frac{\beta}{jk_i} \sum_{\substack{\partial S_m \\ \in C_k}} \int_{\partial S_m} \hat{m}_m \cdot \mathbf{f}_m \sum_{\substack{\partial S_n \\ \in C_{k'}}} (\hat{m}_m \cdot \mathbf{f}_m X_m + \hat{m}_n \cdot \mathbf{f}_n X_n) d\partial S = 0 \quad \text{on } C_{kk'}$$



$$IP_{mn}^i = \frac{\beta}{jk_i} \int_{\partial S_m} \hat{m}_m \cdot \mathbf{f}_m \hat{m}_n \cdot \mathbf{f}_n d\partial S$$

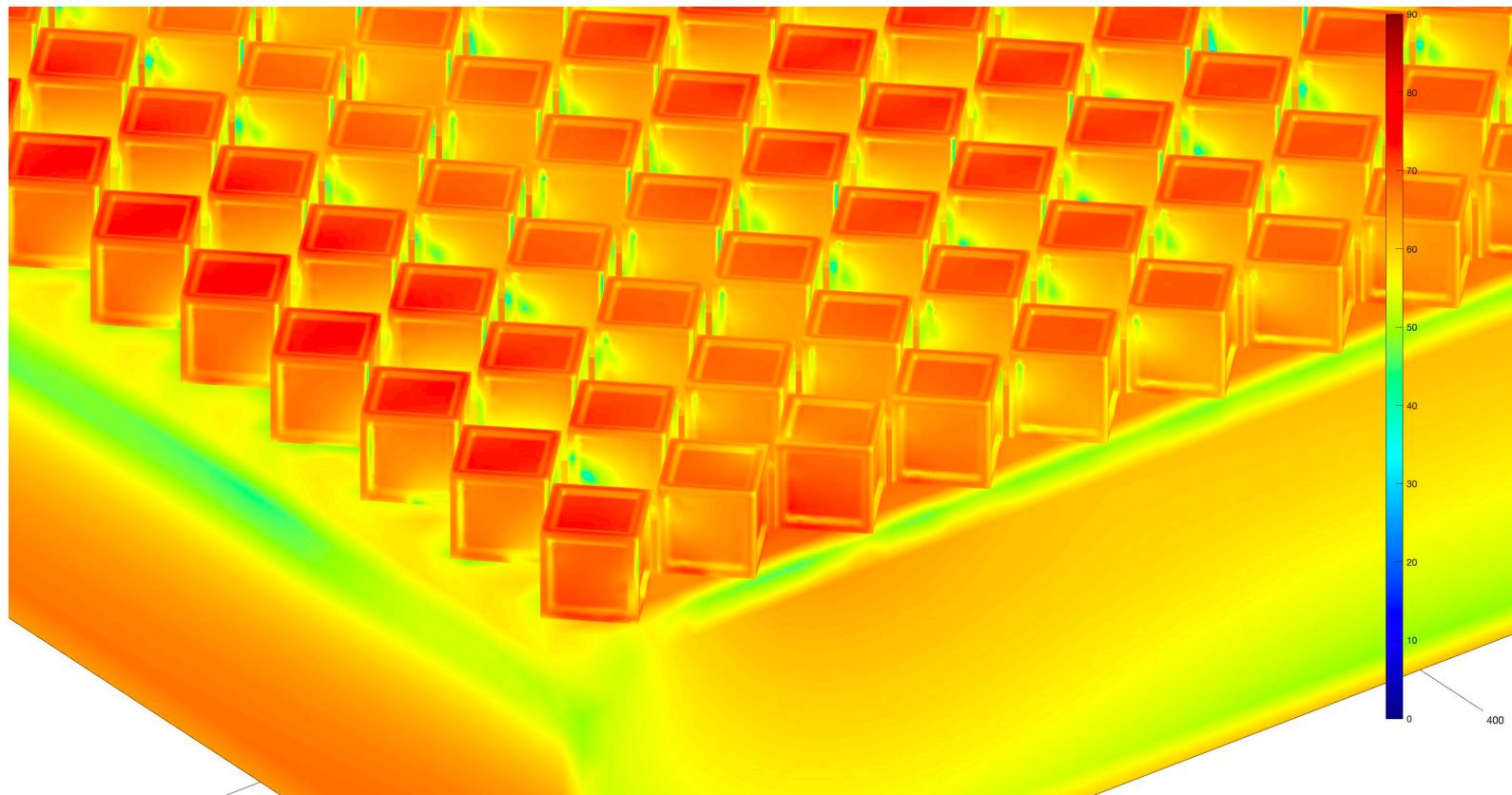
[1] V. F. Martín, L. Landesa, F. Obelleiro, and J. M. Taboada, "A Discontinuous Galerkin Combined Field Integral Equation Formulation for Electromagnetic Modeling of Piecewise Homogeneous Objects of Arbitrary Shape," *IEEE Trans. Antennas Propag.*, vol 70, no 1, pp. 487 - 498 Jan. 2022

# Plasmonic Au nanocubes with Ag depositions over dielectric substrate

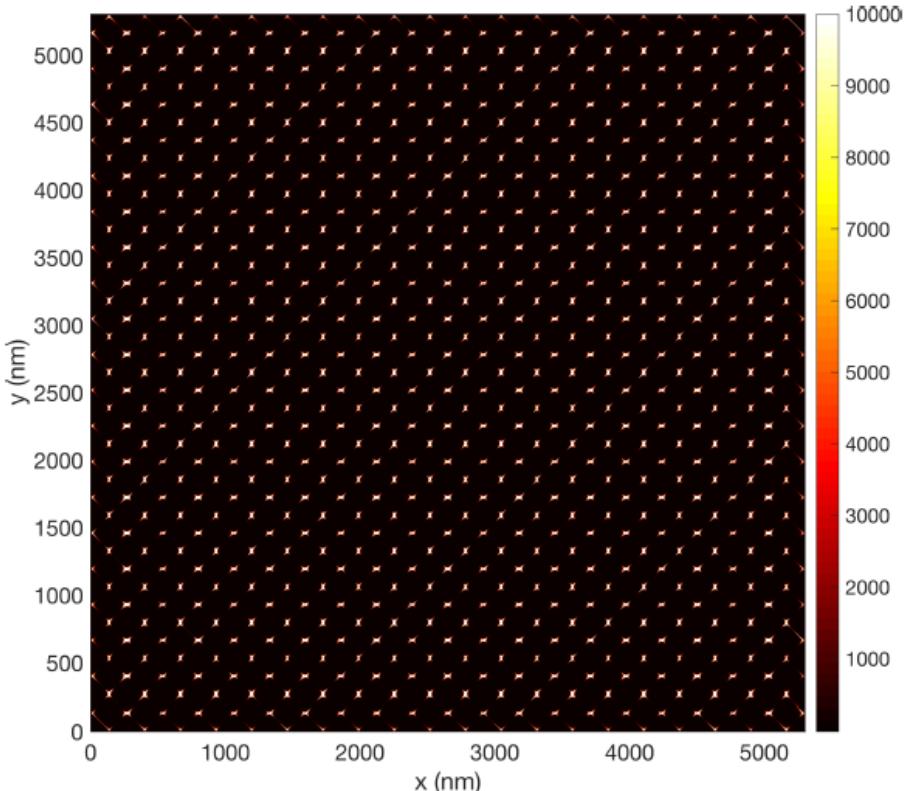
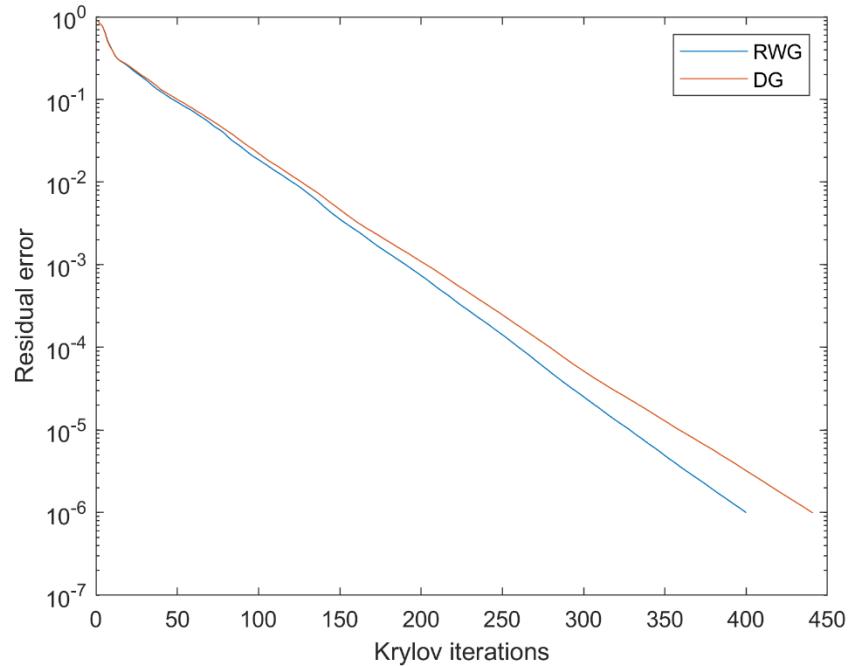


# Plasmonic Au nanocubes with Ag depositions over dielectric substrate

- Equivalent electric currents induced on the external boundary surfaces.



# Convergence



# Multiresolution (MR) Preconditioner

1. Generalized basis functions.

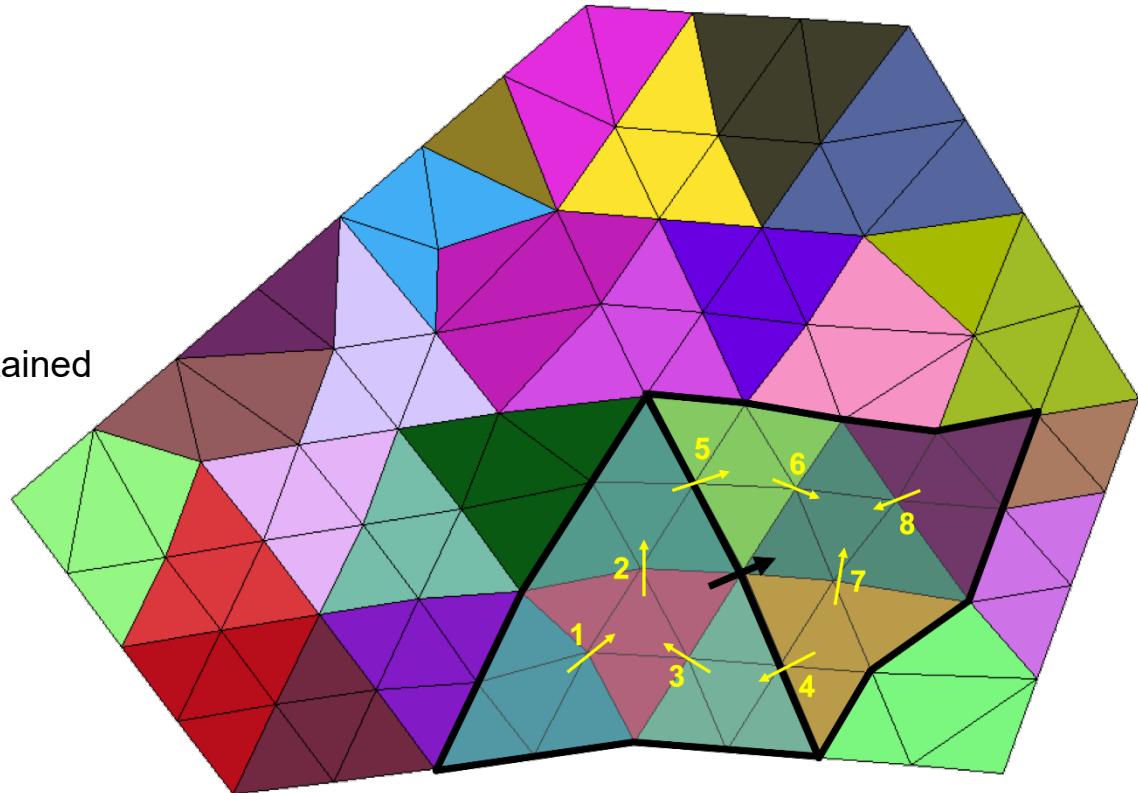
- Reproduce behavior of input RWG.
- Lineal combination of RWG or gRWG.

- $\mathbf{f}_i^l(\mathbf{r}) = \sum_{n=1}^{N_i^{l-1}} f_{i,n}^l \mathbf{f}_{\mu_i^l(n)}^{l-1}(\mathbf{r})$

- $\nabla \cdot$  operator  $[Q_i^l][f_i^l] = [q_i^l]$

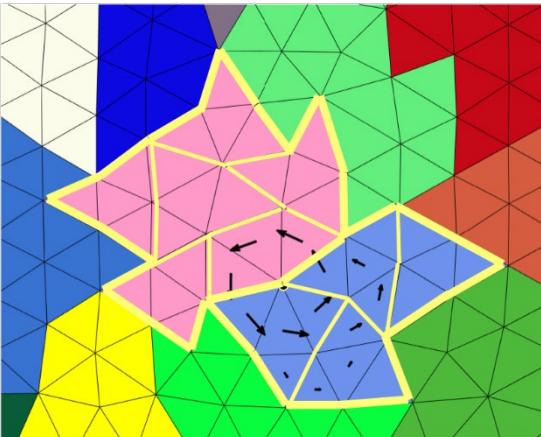
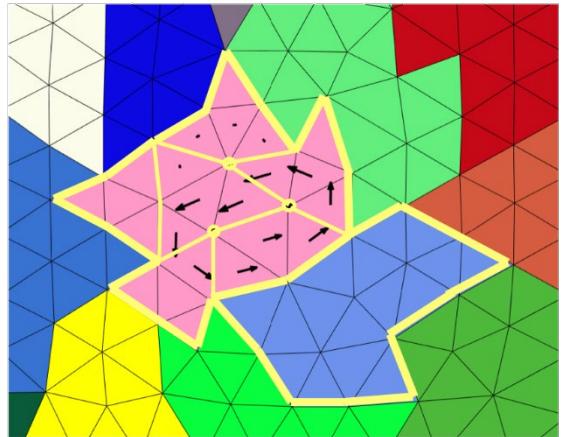
- And the generalized coefficients are obtained

$$\begin{bmatrix} [\hat{Q}_i^l] \\ [U_i^l] \end{bmatrix} [f_i^l] = \begin{bmatrix} [\hat{q}_i^l] \\ [0] \end{bmatrix}$$



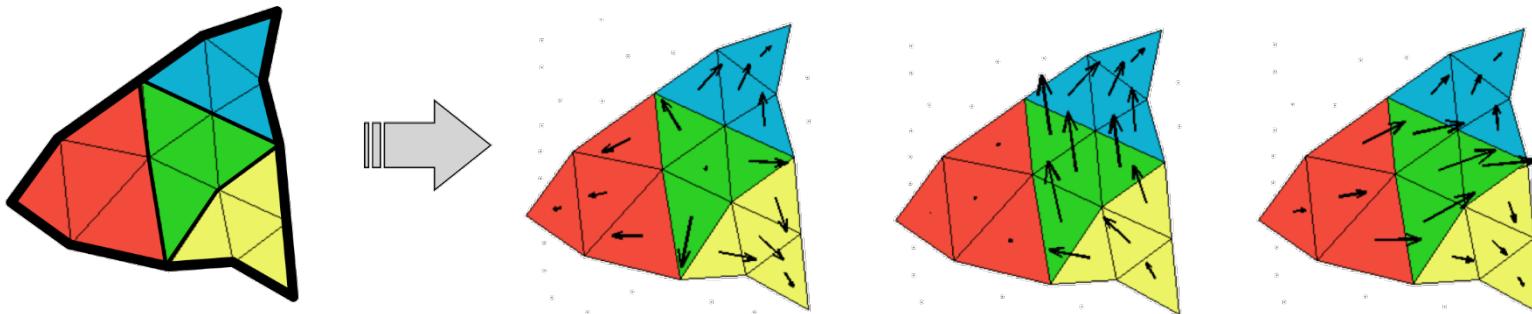
[1] V. F. Martin, J. M. Taboada and F. Vipiana, "A Multi-Resolution Preconditioner for Non-Conformal Meshes in the MoM Solution of Large Multi-Scale Structures," in *IEEE Transactions on Antennas and Propagation*, 2023

# Multiresolution (MR) Preconditioner

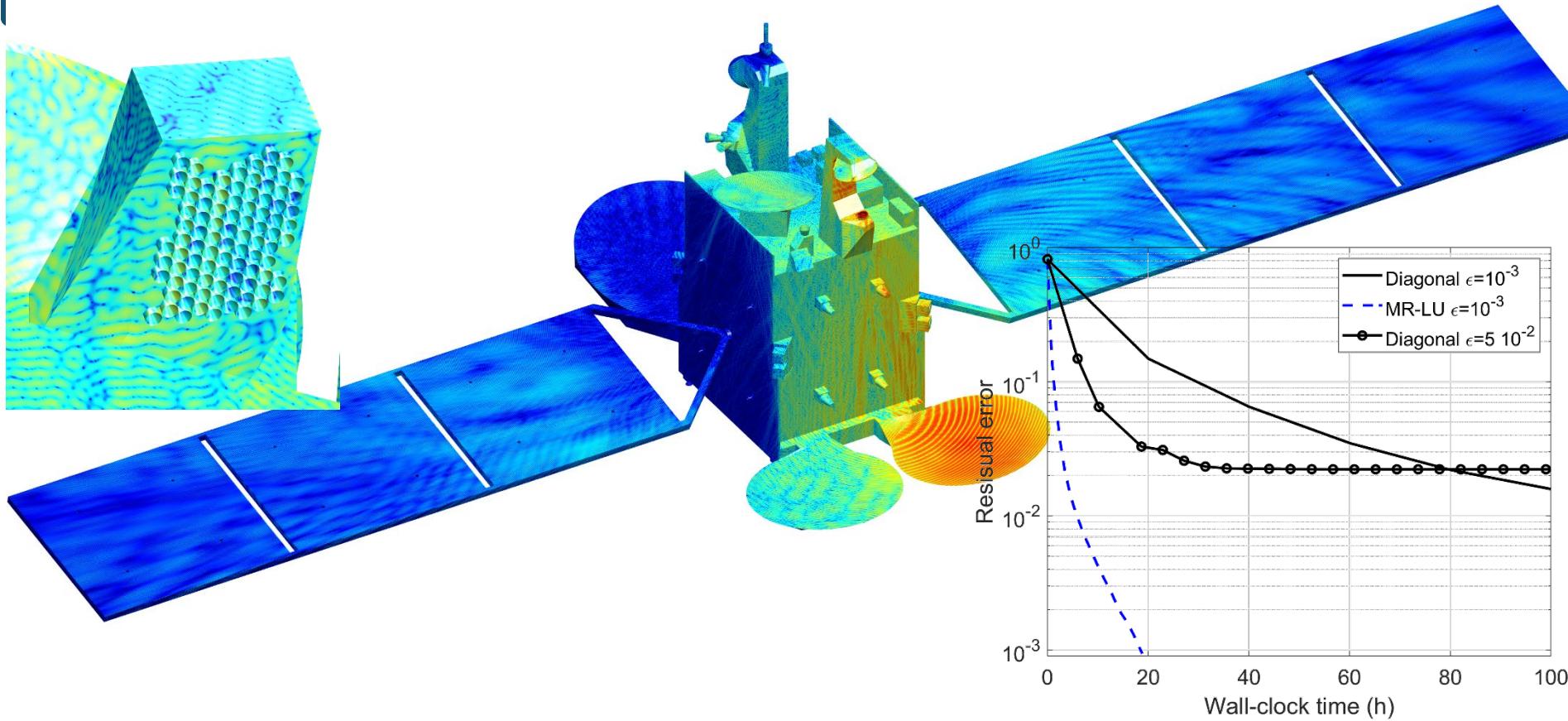


## 3. Multiresolution basis functions.

- Non-solenoidal.
- Solenoidal (1).
- Solenoidal (2).

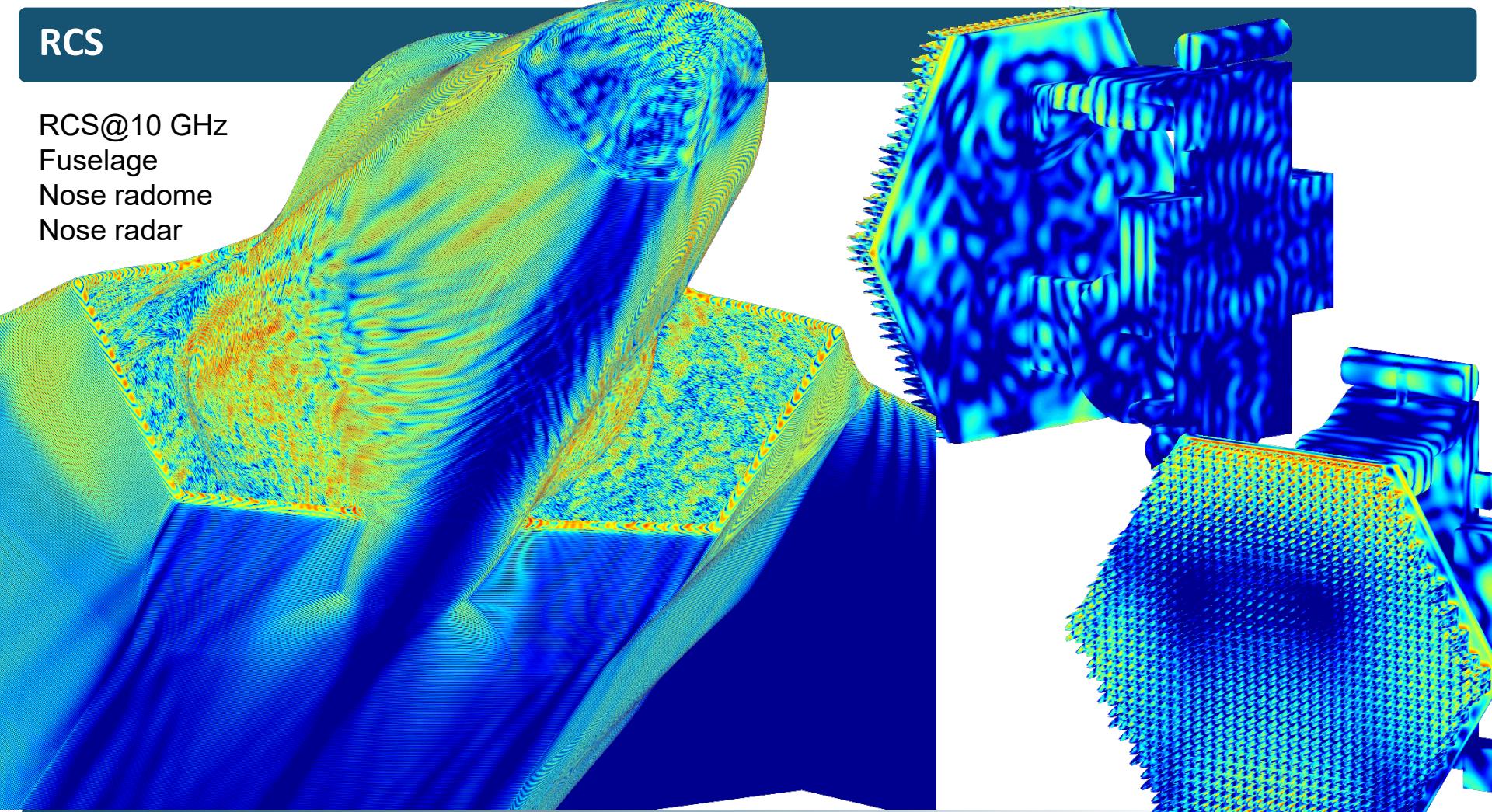


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RCS@10 GHz  
Fuselage  
Nose radome  
Nose radar

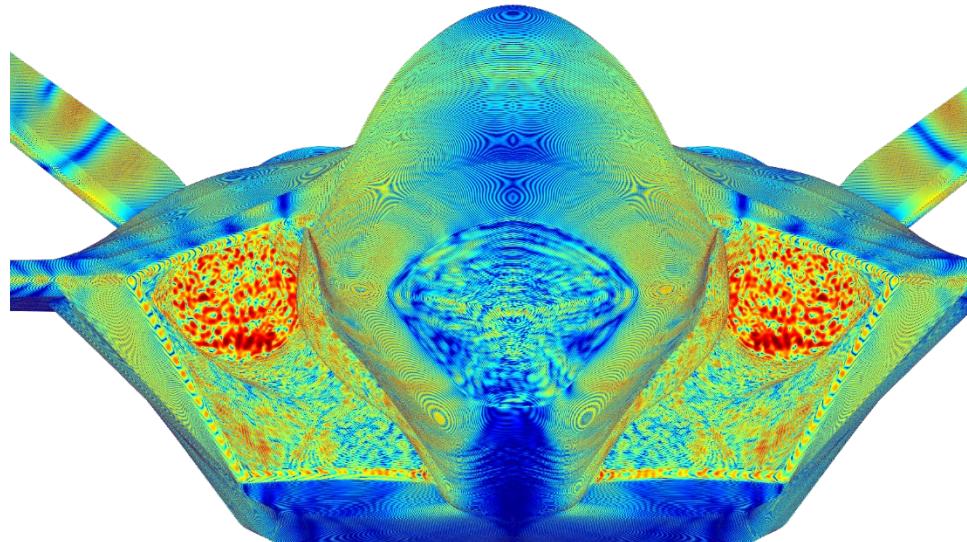


RCS@10 GHz

Fuselage

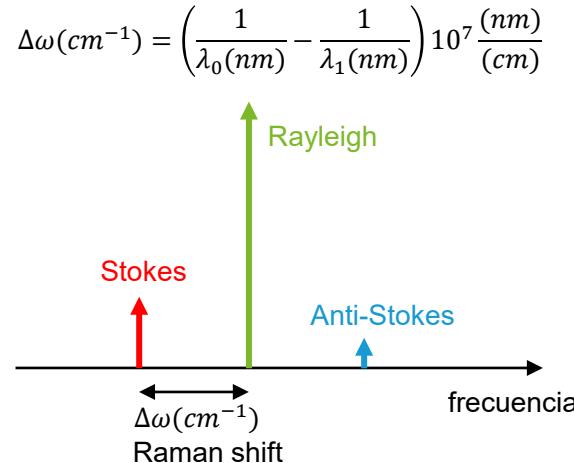
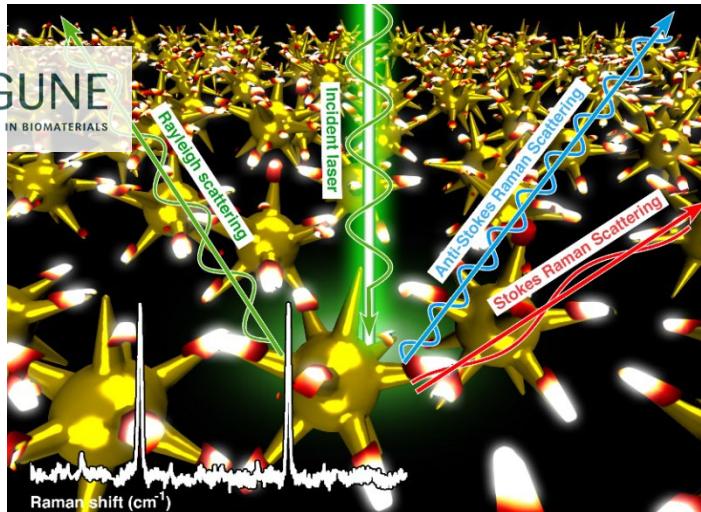
Nose radome

Nose radar



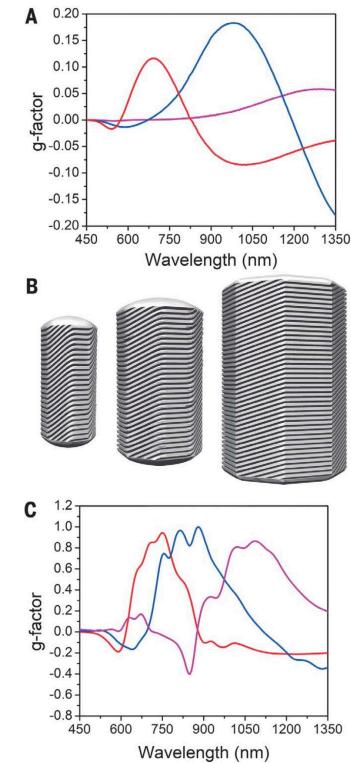
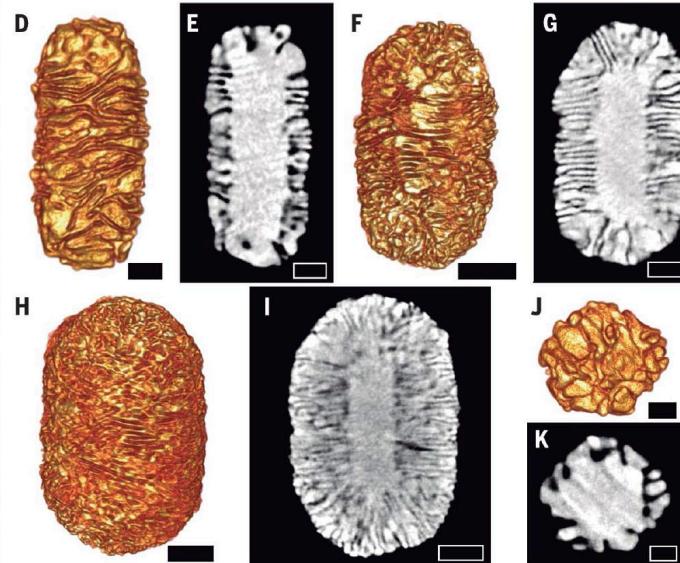
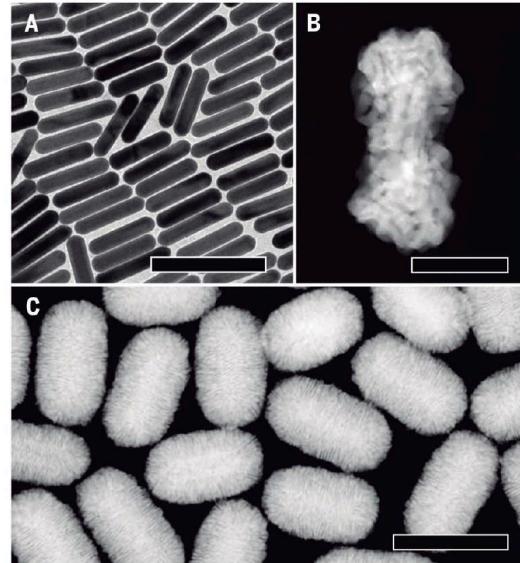
# Plasmonic systems for optical biosensors

- Surface enhanced Raman spectroscopy (SERS): single-molecule detection
- Inelastic (non-linear) Raman scattering: Laser interaction with the molecular vibrational modes → energy shift to blue and red (Raman shift)
- Raman spectrum is like a molecule fingerprint
- Plasmonic substrates can enhance the Raman scattering (SERS) by factors up to  $10^{10}$



[1] D. M. Solís, J. M. Taboada, F. Obelleiro, L. M. Liz-Marzán, and F. J. García de Abajo, "Toward ultimate nanoplasmonics modeling", *ACS Nano*, vol. 8, no. 8, pp. 7559-7570, August 2014.

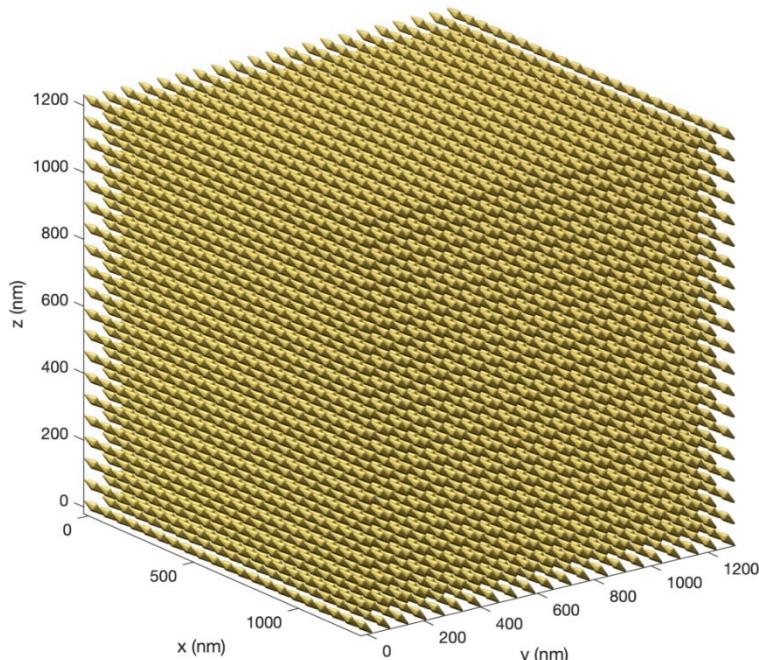
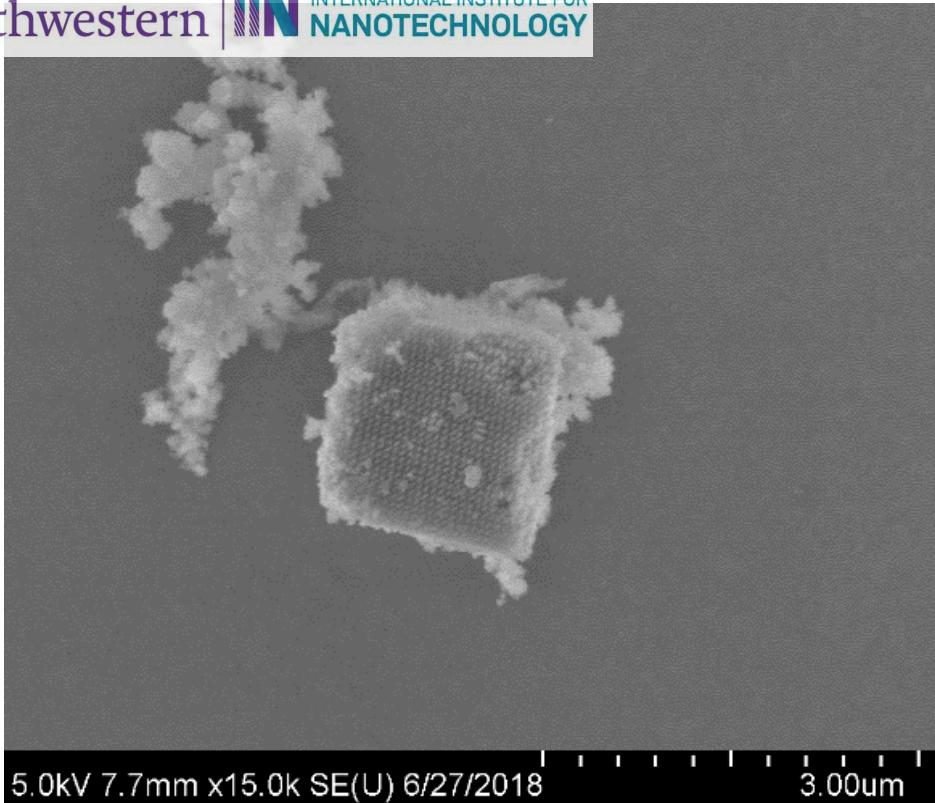
# Chiral nanocrystals



G. González-Rubio, J. Mosquera, V. Kumar, A. Pedrazo-Tardajos, P. Llombart, D. M. Solís, I. Lobato, E. G. Noya, A. Guerrero-Martínez, J. M. Taboada, F. Obelleiro, L. G. MacDowell, S. Bals, L. M. Liz-Marzán, "Micelle-directed chiral seeded-growth on anisotropic gold nanocrystals," *Science*, vol. 368, no. 6498, pp. 1472-1477, June 2020.

# DNA programmed superlattices

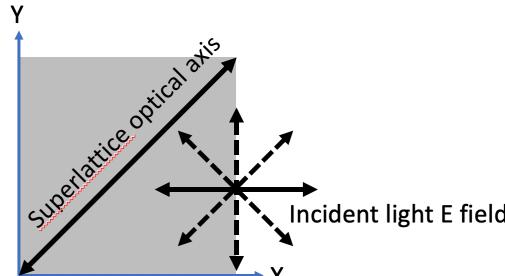
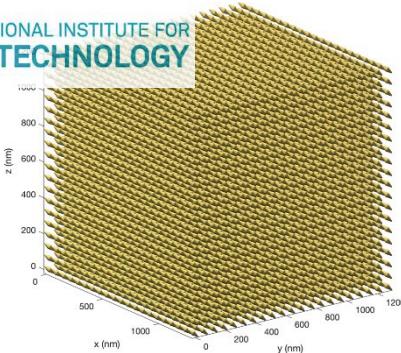
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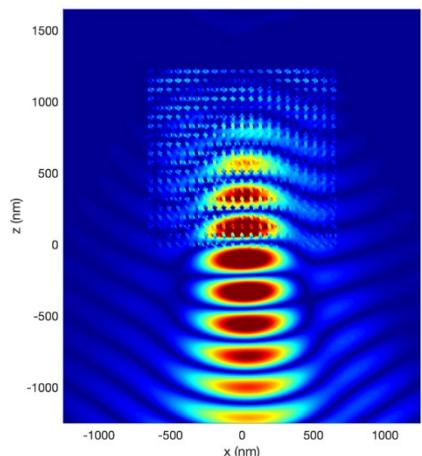
Under review in *Science*

# Anisotropic optical response: birefringence

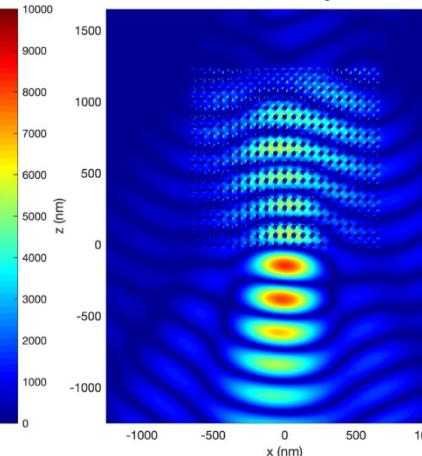
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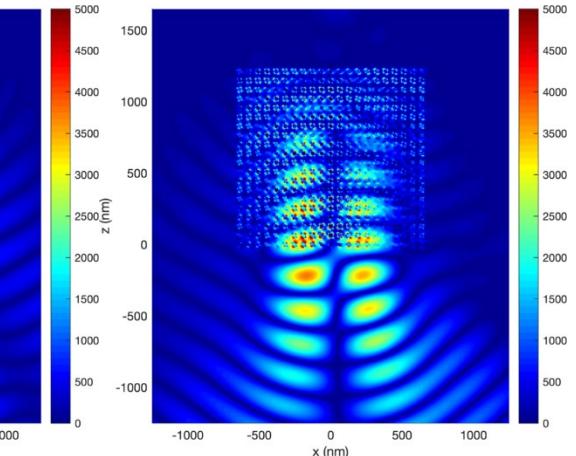
$|\text{Real}(Es_x)|$



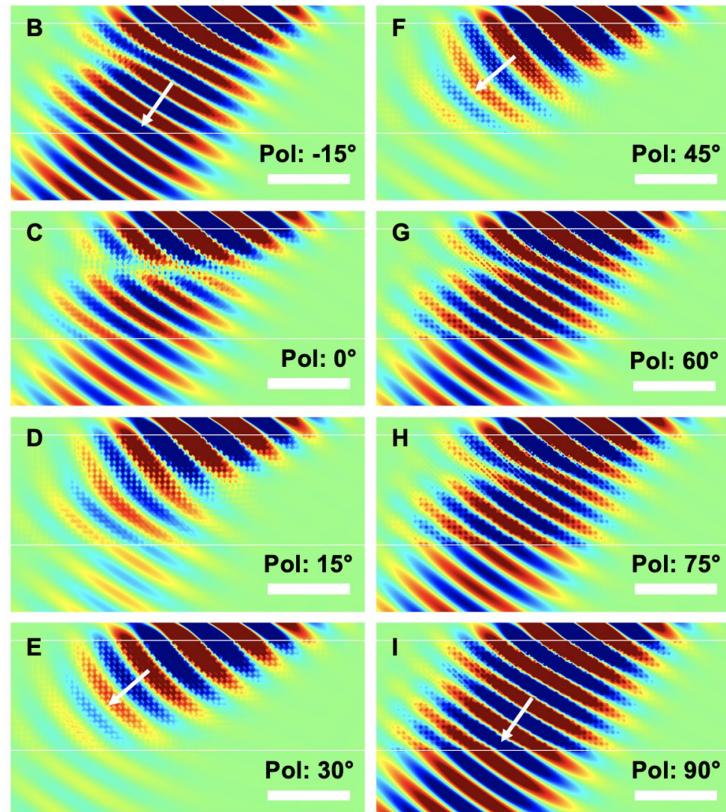
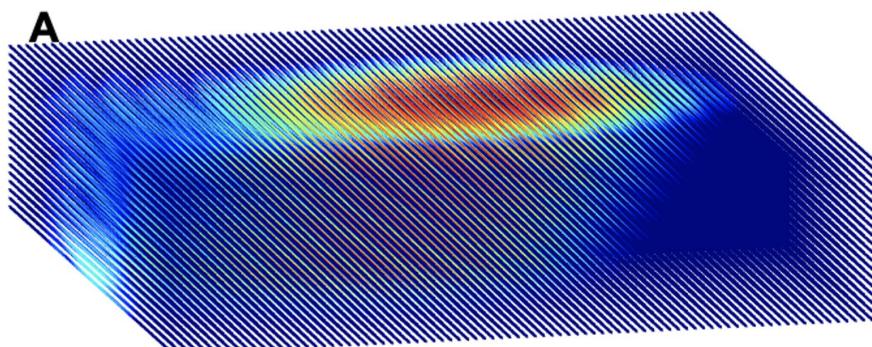
$|\text{Real}(Es_y)|$



$|\text{Real}(Es_z)|$



# Anisotropic optical response: birefringence



# Acknowledgment

This work was supported in part by the Spanish Ministerio de Ciencia, Innovación y Universidades under Project PID2020-116627RB-C21, Project PID2020-116627RB-C22, funded by MCIN/AEI/10.13039/501100011033 and grant FPU00550/17, and the Extremadura local government and European Union FEDER (GR18055, IB18073).



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# Anisotropic optical response: birefringence

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