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Cherenkov emission in a nanowire material

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Outline

- Introduction
- Motivation
- Cherenkov Radiation Characterization Model
- Stopping Power: Boosting the Radiation Emission
- Density of States
- Conclusions

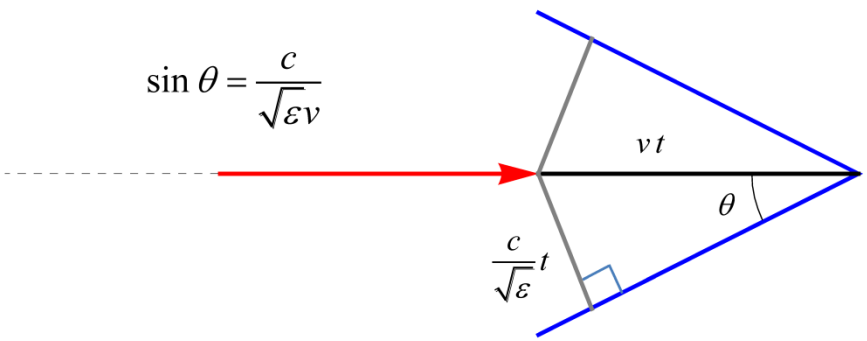
Introduction

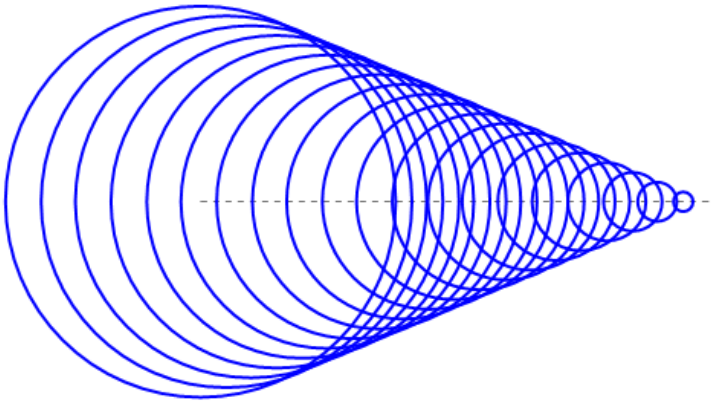
The Cherenkov Effect

- Appears when charged particle moves at a velocity ***v*** larger than the phase velocity of the electromagnetic waves inside the medium $\boldsymbol{v_{ph}}$ → **Velocity Threshold!**

- Emitted radiation beam consists of conical wave fronts with aperture 2θ:

$$\sin\theta = \frac{c}{\sqrt{\varepsilon}v}$$

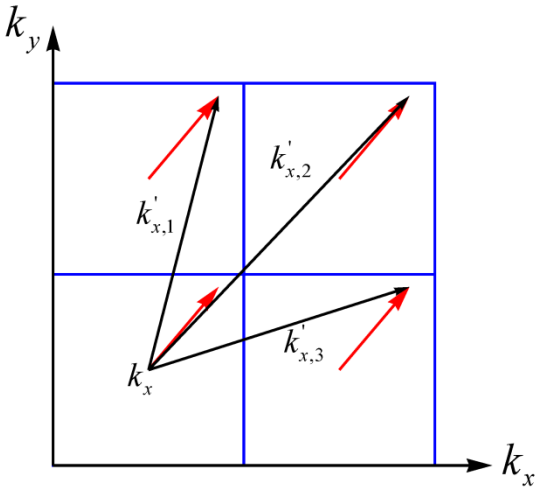




- Particle detection in high energy physics.

Motivation

Motivation for studying periodic structures in the context of the Cherenkov effect



- Umklapp processes: Periodicity of the wave number

$$k'_{x,n} = k_x + 2\pi n / a \qquad k_x = \omega / v$$

- Appears a possibility of matching with $k_0 = \omega / c$ for low velocities.

- Possibility of absence of velocity Threshold!
  - May allow for the detection of particles moving at low velocities (not possible with conventional dielectric structures).

Nanowire Material: Definition

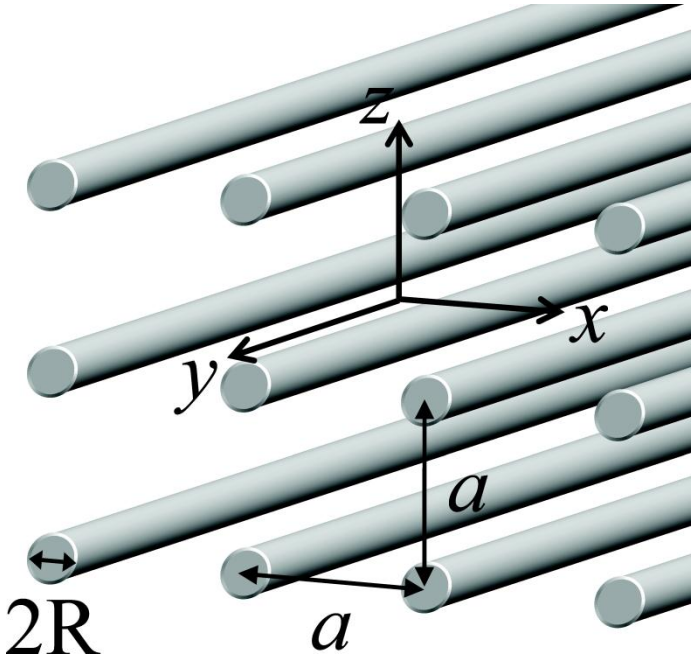
- 2D array of conducting wires embedded in a host material.

- The homogenized wire medium is described by an effective dielectric permittivity tensor:

$$\overline{\overline{\varepsilon}} = \begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix}$$

$$\varepsilon_{xx} = \varepsilon_{zz} = 1$$

$$\varepsilon_{yy} = 1 + \left[\frac{\varepsilon_h}{(\varepsilon_m - \varepsilon_h) f_V} - \frac{\beta_h^2 - k_y^2}{\beta_p^2} \right]^{-1}$$



- Granularity is neglected in the effective medium approach → Time dispersion in effective permittivity is present.

$$\beta_p^2 \approx 2\pi \Big/ \left[a^2 \left(0.5275 + \ln\left(a/2\pi R \right) \right) \right]$$

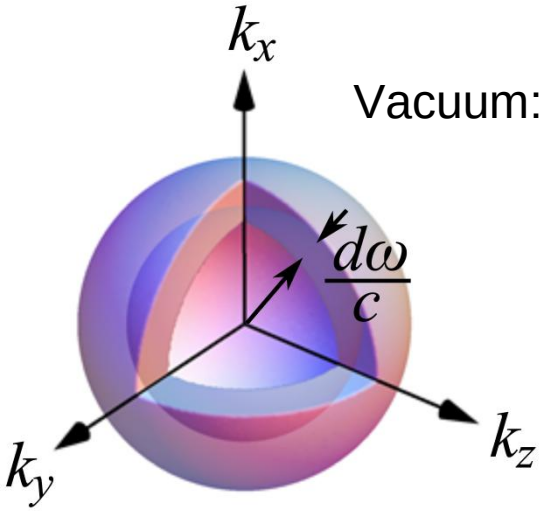
$$\beta_h = \beta\sqrt{\varepsilon_h} = \sqrt{\varepsilon_h}\,\omega/c$$

Nanowire Material: Why this periodic structure?

- Ultrahigh density of states → Larger number of radiative decay channels.
  - Density of states proportional to the volume → Related to the number of wires!

Boosts the interaction between matter and radiation at low velocities!

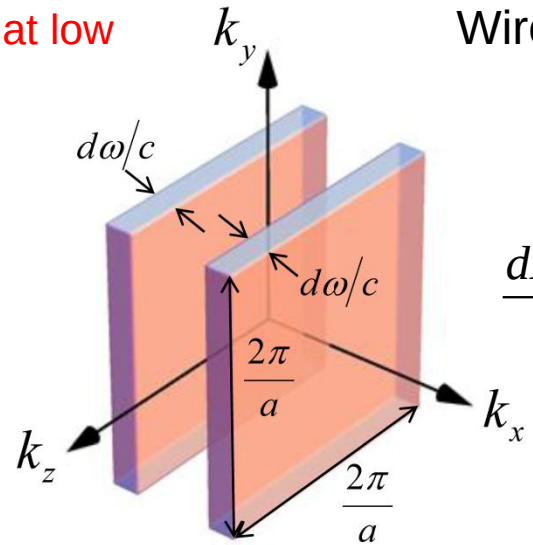
Vacuum:



$$\left(\frac{\omega}{c}\right)^2 = k_x^2 + k_y^2 + k_z^2$$

$$\frac{dN_{TM}}{V_0} = \frac{\omega^2}{2\pi^2 c^3} d\omega$$

Wire-Medium:



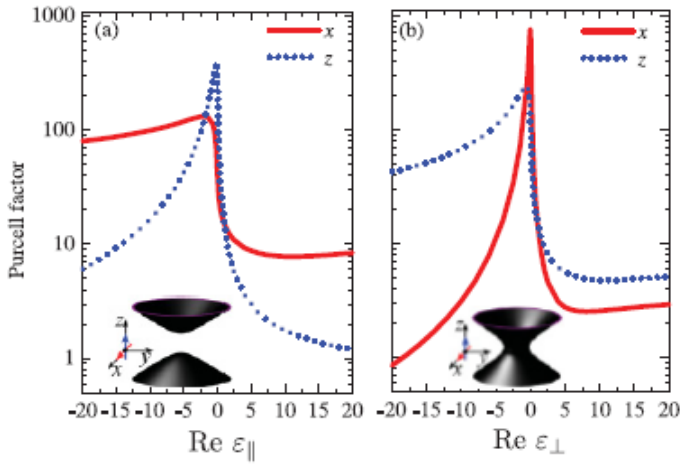
$$\left(\frac{\omega}{c}\right)^2 = k_x^2$$

$$\frac{dN_{TEM}}{V_0} = \frac{1}{\pi a^2 c} d\omega$$

S. I. Maslovski , M. G. Silveirinha, *Phys. Rev. A*, **82**, 022511 (2011).

Nanowire Material: Why this periodic structure?

- Media with hyperbolic dispersion → Enhancement of spontaneous emission .



Alexander N. Poddubny , Pavel A. Belov, Yuri S. Kivshar, *Phys. Rev. A*, **84**, 023807(2011)

- Possibility to guide radiation in a specific direction.

- Cherenkov radiation in wire-medium structures.

David E. Fernandes, Stanislav I. Maslovski , Mário G. Silveirinha, *Phys. Rev. B*, **85**, 155107 (2012)

Viktor V. Vorobev ,Andrey V. Tyukhtin, *Phys. Rev. Lett.*, **108** 184801 (2012)

Cherenkov Radiation Characterization Model

Studied Configuration

- Linear array of charges moving at a constant velocity ***v*** confined to the plane ***y=0.***

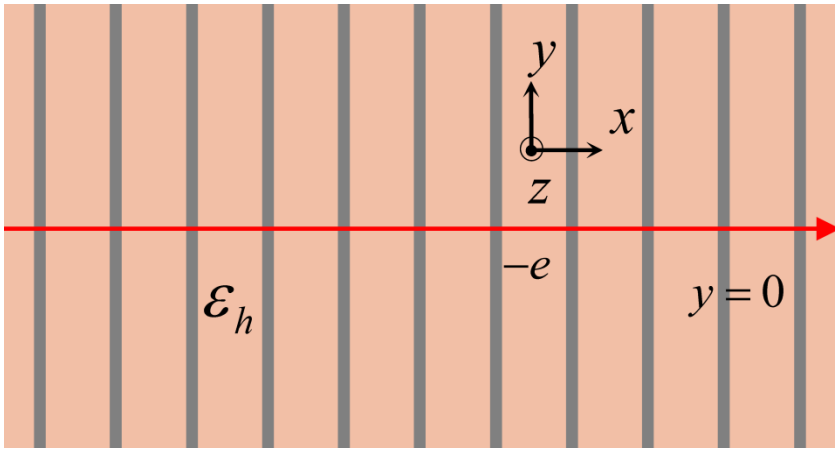
- Current density:

$$\mathbf{J}(x,y,t) = -en_z\delta(y)\delta(x/v - t)\hat{\mathbf{x}}$$

$$\Big\downarrow F.T.$$

$$\mathbf{J}(x,y,\omega) = -en_z\delta(y)e^{-jk_x x}\hat{\mathbf{x}}$$

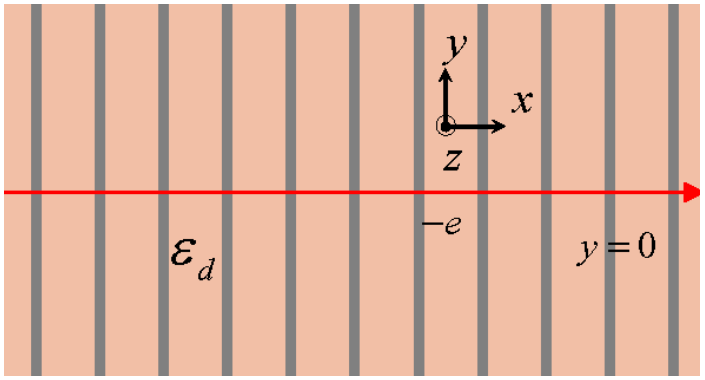
$$k_x = \omega/v$$



Field Characterization

- The solution of the Maxwell equations characterizes the interaction between the charges and the structure:

$$\nabla \times \boldsymbol{H} - \partial \boldsymbol{D}/\partial t = \boldsymbol{J} \qquad \partial \boldsymbol{B}/\partial t + \nabla \times \boldsymbol{E} = 0$$



- In the frequency domain the general solution of the problem is : $\mathbf{H} = H_z \hat{\mathbf{z}}$

$$H_z(x,y,\omega) = \begin{cases} \left(T_{TEM}^{+} e^{-\gamma_{TEM} y} + T_{TM}^{+} e^{-\gamma_{TM} y}\right) e^{-jk_x x} & y > 0 \\ \left(T_{TEM}^{-} e^{\gamma_{TEM} y} + T_{TM}^{-} e^{\gamma_{TM} y}\right) e^{-jk_x x} & y < 0 \end{cases}$$

Superposition of 2 modes

$$\gamma_{TEM} = j\left[\beta_h^2 - \frac{1}{2}\left(\beta_p^2 + k_t^2 - \beta_c^2 - \sqrt{\left(\beta_p^2 + k_t^2 - \beta_c^2\right)^2 + 4k_t^2\beta_c^2}\right)\right]^{\frac{1}{2}} \qquad \gamma_0 = \sqrt{k_x^2 - \beta^2}$$

$$\gamma_{TM} = j\left[\beta_h^2 - \frac{1}{2}\left(\beta_p^2 + k_t^2 - \beta_c^2 + \sqrt{\left(\beta_p^2 + k_t^2 - \beta_c^2\right)^2 + 4k_t^2\beta_c^2}\right)\right]^{\frac{1}{2}}$$

Field Pattern

•Solution of the scattering problem:

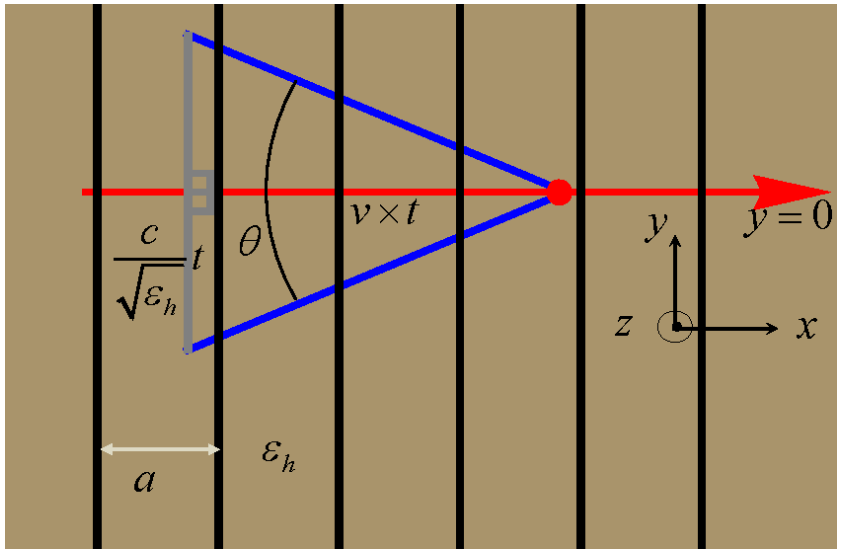
$$H_z(x,y,\omega)=\frac{e\,n_z}{2}\operatorname{sgn} y\left(\frac{\varepsilon_h\omega^2/c^2+\gamma_{TM}^2-k_x^2}{\gamma_{TEM}^2-\gamma_{TM}^2}e^{-\gamma_{TEM}|y|}-\frac{\varepsilon_h\omega^2/c^2+\gamma_{TEM}^2-k_x^2}{\gamma_{TEM}^2-\gamma_{TM}^2}e^{-\gamma_{TM}|y|}\right)e^{-jk_x x}$$

•It is expected a strong coupling of the EMF inside the nanowire structure *(coupling wire-by-wire).*

•TEM waves travel in direction parallel to the wires at a velocity $v_{TEM}=c/\varepsilon_h$

•Well defined radiation pattern:

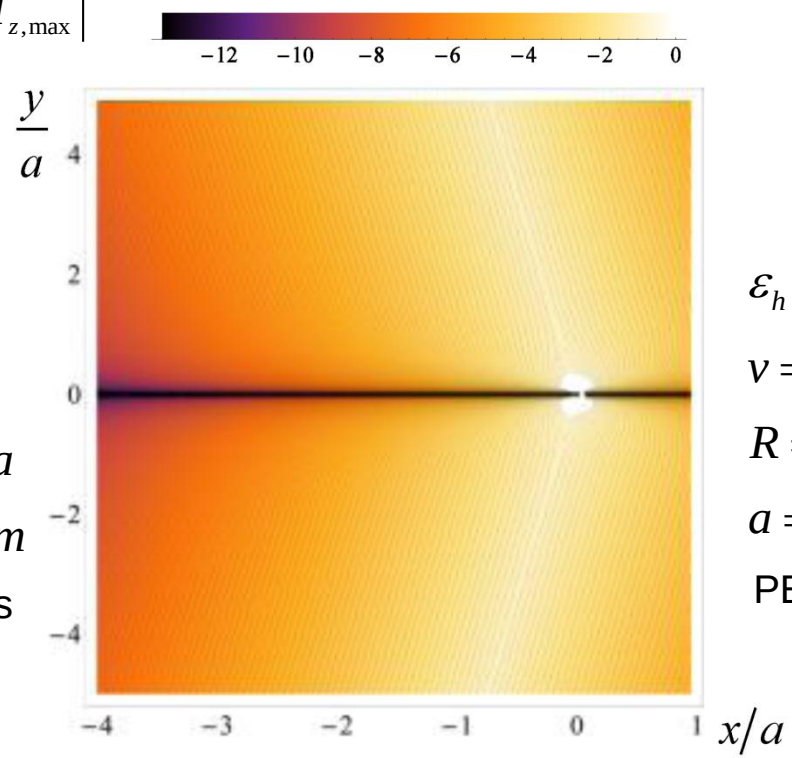
$$\tan\theta=\frac{c}{\sqrt{\varepsilon_h}v}$$



Electromagnetic Field Distribution

- **Snapshot of the magnetic field:**
  - Charges are moving for a long time and source of charges is far away.
  - Snapshot of the Magnetic Field at t=0 is enough to characterize Cherenkov emission.

$$\mathrm{Log}\left|\frac{H_z}{H_{z,\max}}\right|$$



$\varepsilon_h = 1$

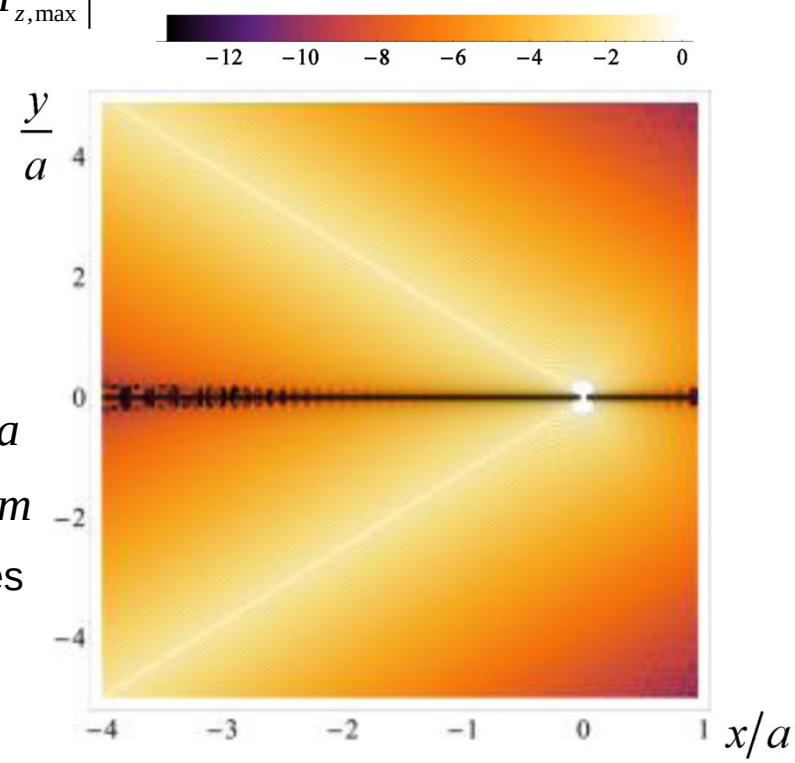
$v = 0.15c$

$R = 0.05a$

$a = 100nm$

PEC wires

$$\mathrm{Log}\left|\frac{H_z}{H_{z,\max}}\right|$$



$\varepsilon_h = 1$

$v = 0.8c$

$R = 0.05a$

$a = 100nm$

PEC wires

Electromagnetic Field Distribution

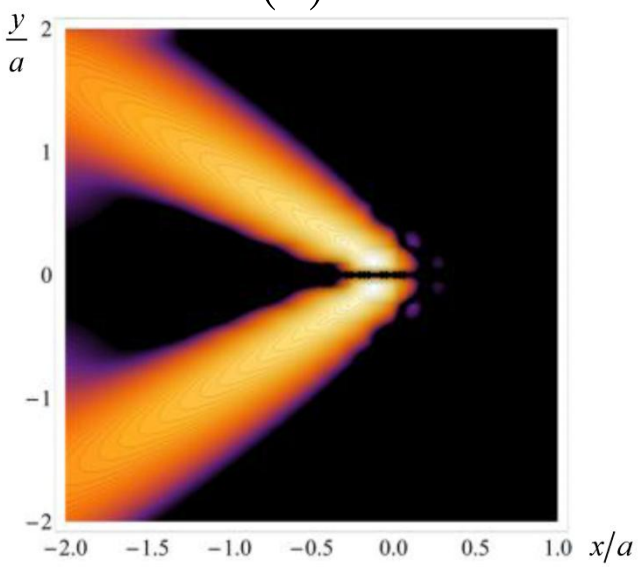
$$\varepsilon_h(\omega) = \varepsilon_\infty + \frac{\varepsilon(0) - \varepsilon_\infty}{1 + j\omega\tau}$$

$$v = 0.5c \qquad R = 0.05a$$

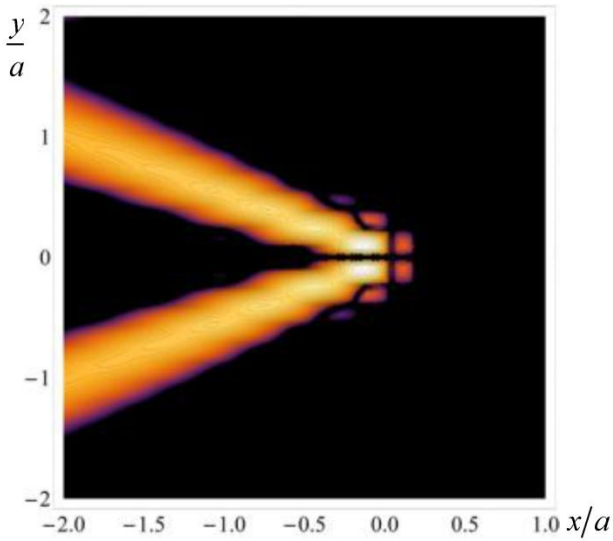
$$\tau = 0.33[\text{fs}] \quad \varepsilon_\infty = 1$$

$$\theta = \tan^{-1} c / \left(v\sqrt{\varepsilon_h}\right)$$

$\varepsilon(0) = 10$



$\varepsilon(0) = 20$



- As ε(0) increases the angle of the of the radiation pattern decreases.
- There is no longer radiation ahead of the position of the charges.

Electromagnetic Field Distribution

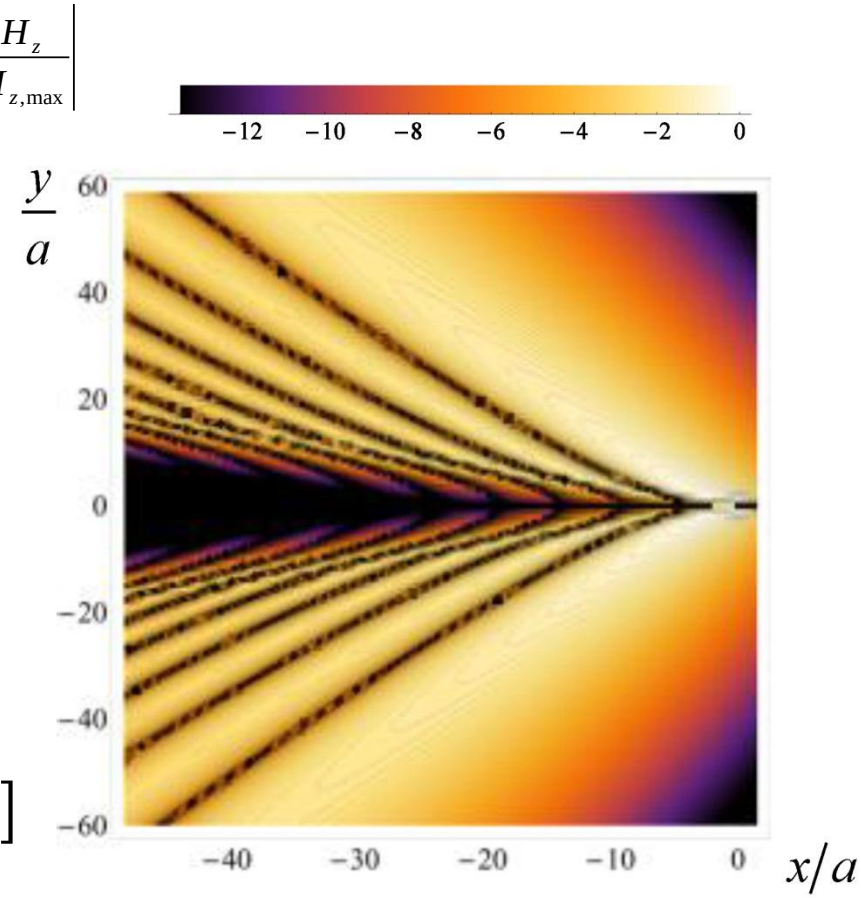
- Interference pattern → propagation constants depend on k_x .

- Beam is still very directive: $\theta \sim \tan^{-1} c / \left(v\sqrt{\varepsilon_h} \right)$

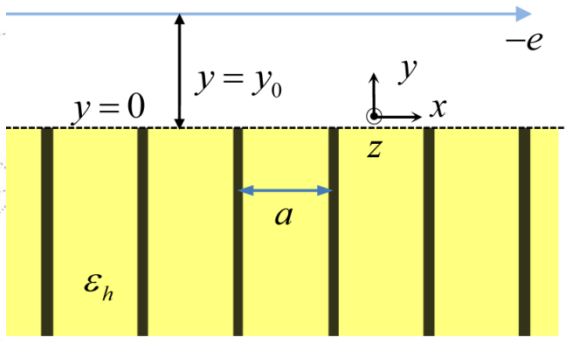
$$\varepsilon_m(\omega) = 1 - \omega_p^2 / \omega(\omega - j\Gamma) \qquad \varepsilon_h = 1$$

$$\Gamma / 2\pi = 4.35\,[\mathrm{THz}] \qquad \omega_p / 2\pi = 2175\,[\mathrm{THz}]$$

$$R = 0.05a \qquad a = 100\,nm$$



Electromagnetic Field Distribution



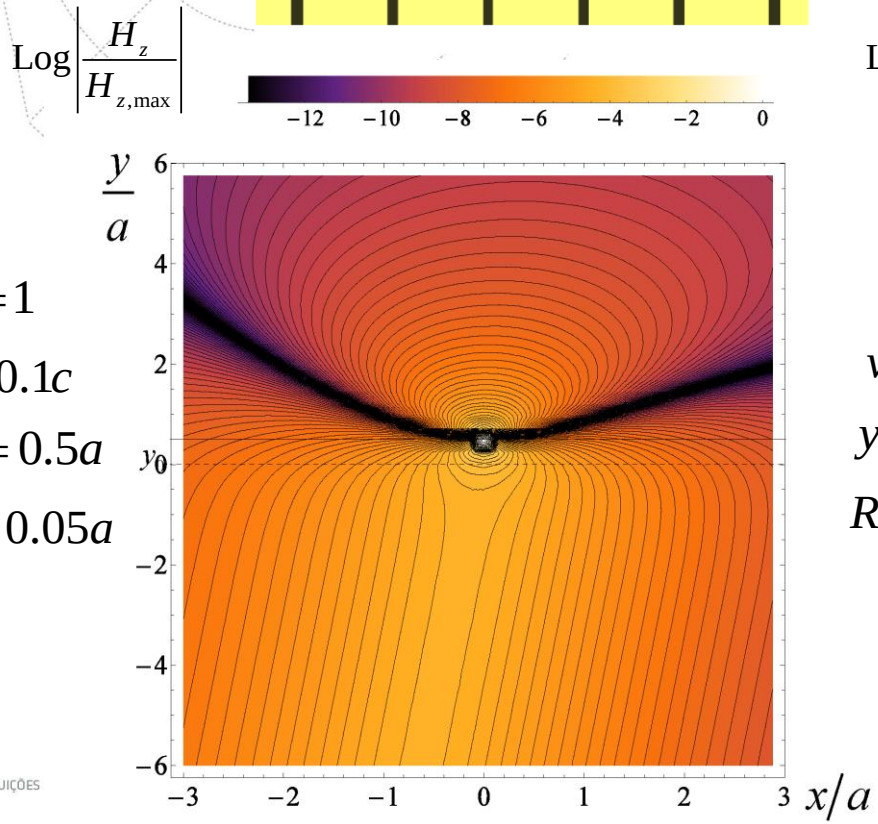
- No direct interaction between charges and Wire-Medium → No bremsstrahlung radiation!!

$\varepsilon_h = 1$

$v = 0.1c$

$y_0 = 0.5a$

$R = 0.05a$

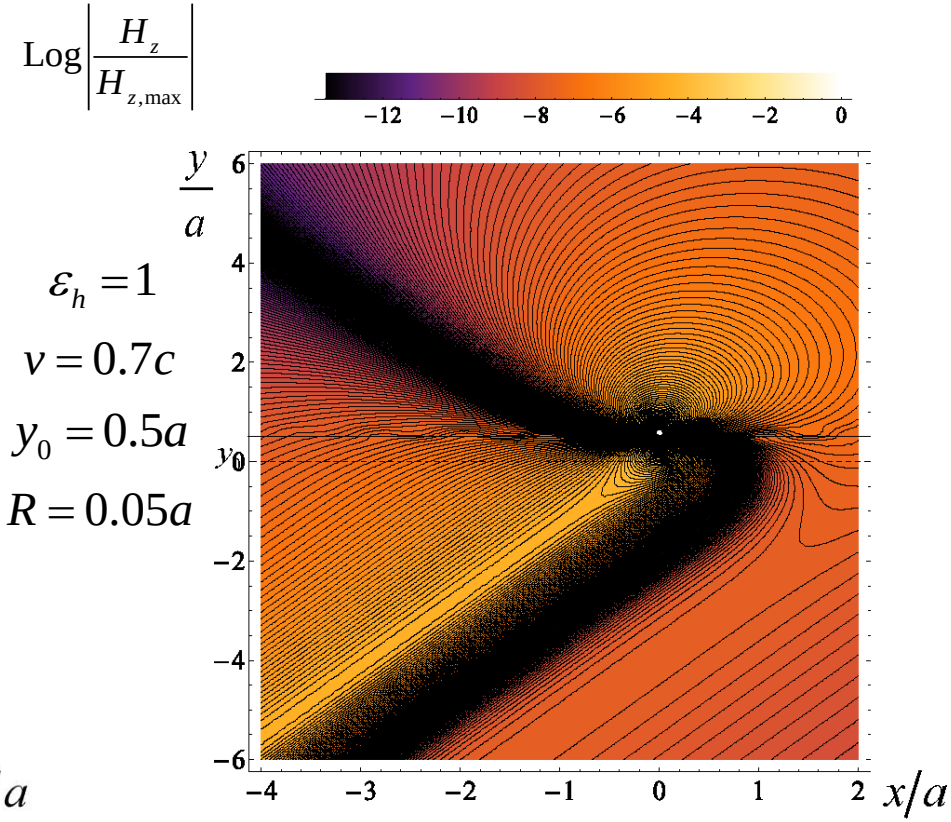


$\varepsilon_h = 1$

$v = 0.7c$

$y_0 = 0.5a$

$R = 0.05a$



Stopping Power

Stopping Power Integral

- **Our approach:**
  - Emitted radiation causes energy loss.
  - Charges remain at constant velocity due to an external force.

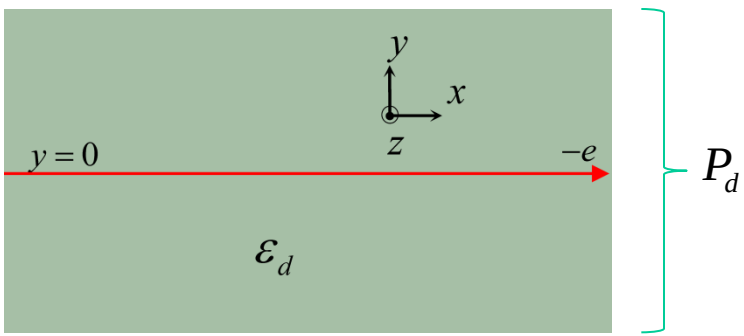
- We may calculate the stopping power through the integral:

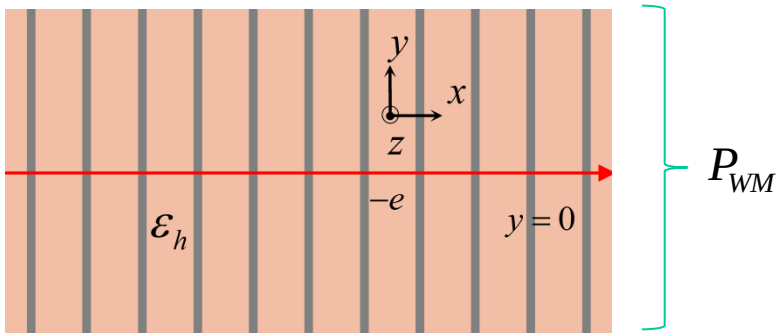
$$P_0 = -\int \mathbf{E}_{loc} \cdot \mathbf{J}\, d^3r$$

Our configuration: x-component of the local field that acts on the charges

- Instantaneous power provided to each charge is:

$$P_0 / N_z = evE_{loc,x}\left(vt, y_0, t\right)$$





Comparison of the values of the stopping power

- Ratio between the stopping powers of the wire-medium and of the dielectric material (water) P_{WM}/P_d :

$$\varepsilon_d(\omega) = \varepsilon_\infty + \frac{\varepsilon(0) - \varepsilon_\infty}{1 + j\omega\tau}$$

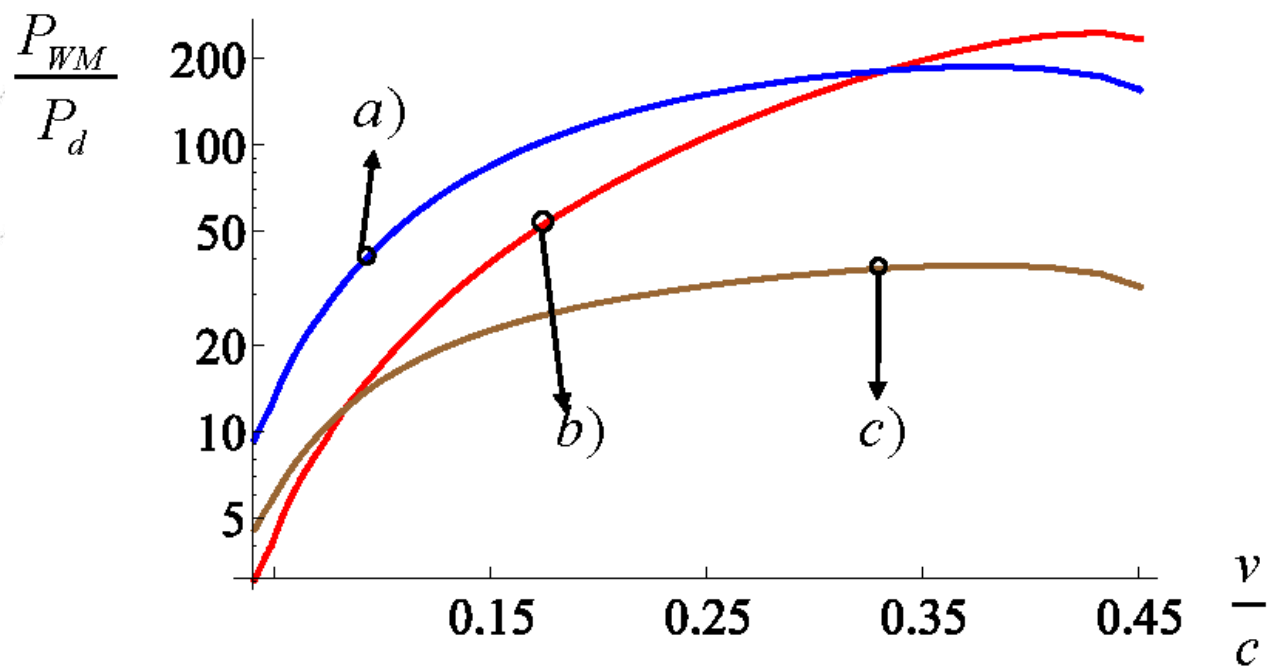
$$R = 0.05a$$

$$T = 20^{\circ}C$$

$$\varepsilon(0) = 80.21$$

$$\tau = 9.36[\text{ps}]$$

$$\varepsilon_\infty = 5.6$$



a) Silver Nanowires and host is vacuum

b) PEC Nanowires and host is vacuum

c) Silver Nanowires and host is water

$$\varepsilon_\infty \neq 1 \Rightarrow v < \frac{c}{\sqrt{\varepsilon_h}} \approx 0.48c$$

Comparison of the values of the stopping power

- Ratio between the stopping powers of the semi-infinite wire-medium and of the semi-infinite dielectric material (water) P_{WM}/P_d :

$$\varepsilon_d(\omega) = \varepsilon_\infty + \frac{\varepsilon(0) - \varepsilon_\infty}{1 + j\omega\tau}$$

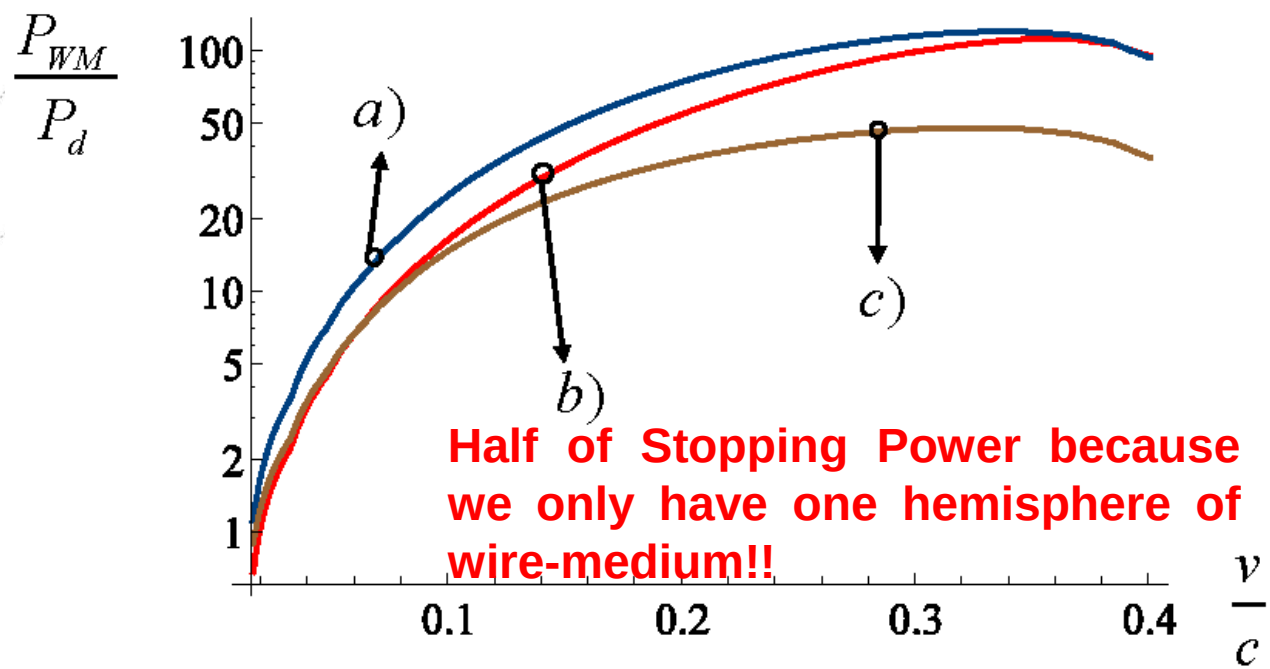
$$R = 0.05a$$

$$T = 20^{\circ}C$$

$$\varepsilon(0) = 80.21$$

$$\tau = 9.36[\text{ps}]$$

$$\varepsilon_\infty = 5.6$$



Half of Stopping Power because we only have one hemisphere of wire-medium!!

a) Silver Nanowires and host is vacuum

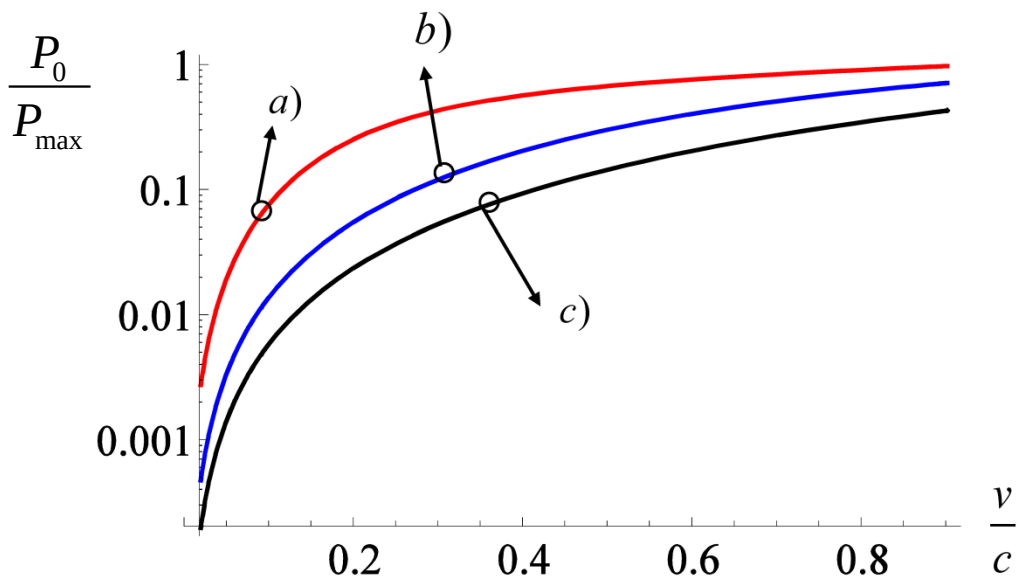
b) PEC Nanowires and host is vacuum

c) Silver Nanowires and host is water

$$\varepsilon_\infty \neq 1 \Rightarrow v < \frac{c}{\sqrt{\varepsilon_h}} \approx 0.48c$$

Density of States

- **Effect of density of states in the value of stopping power:**
  - As seen before the number of photonic states is independent of the frequency and varies with $1/a^2$.



$a)\ a = 100\ nm \quad b)\ a = 300\ nm \quad c)\ a = 600\ nm$

  - As the lattice period decreases the stopping power increases!

Conclusions

Conclusions

- **The nanowire material allows for Cherenkov emission with no velocity threshold.**
- **The stopping power characteristic of the nanowire material can be more than 200 times larger than in other more conventional structures such as dielectrics.**
- **The enhancement of the amount of emitted Cherenkov radiation is due to the extremely large density of photonic states.**
- **Our study may have interesting applications in the context of particle detection based on the Cherenkov phenomena.**

David E. Fernandes, Stanislav I. Maslovski , Mário G. Silveirinha, *Phys. Rev. B*, **85**, 155107 (2012)

Thank you for your time!

Thank you for your time!

Thank you for your time!

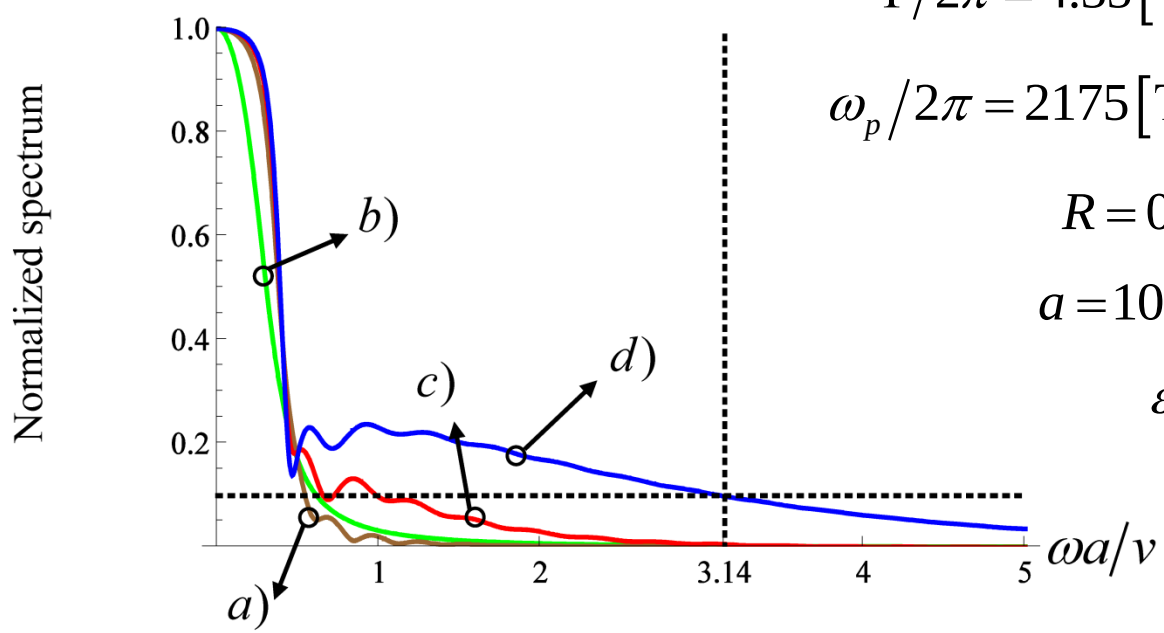
- **Maximum charge velocity :**
  - At the edge of the first Brillouin zone the amplitude of the magnetic field should be less than 10% of it's maximum value.

$a)\ v = 0.3c$

$b)\ v = 0.7c$

$c)\ v = 0.9c$

$d)\ v = 0.98c$



$$\Gamma/2\pi = 4.35\,[\mathrm{THz}]$$

$$\omega_p/2\pi = 2175\,[\mathrm{THz}]$$

$$R = 0.05a$$

$$a = 100\ nm$$

$$\varepsilon_h = 1$$

- If silver nanowires are embedded in a vacuum medium our model is valid to velocities up to v≈0.98c!!

- **Maximum integration region in stopping power calculations:**
  - Maximum frequency of integration → maximum value of k_x

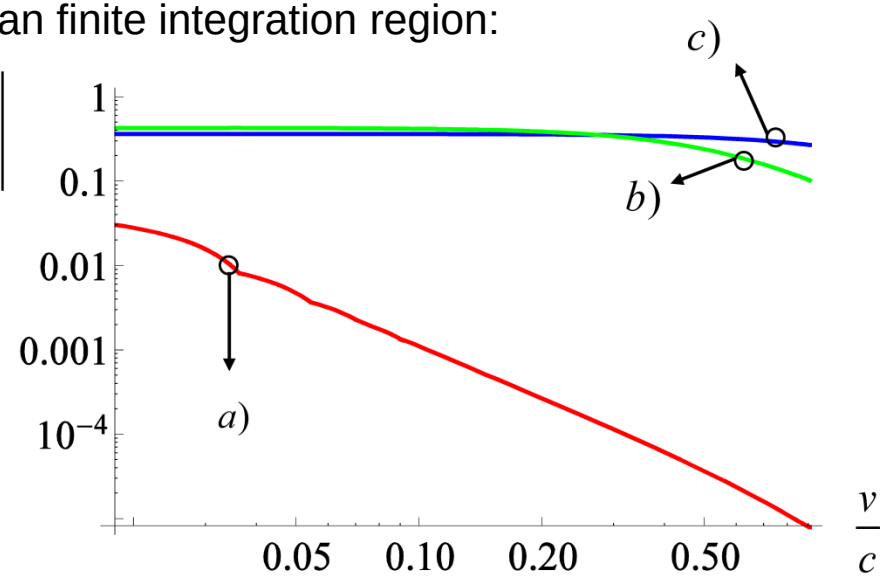
$$|k_x| < k_{\max} \approx \frac{\pi}{a} \qquad k_x = \omega / v \qquad \omega < \omega_{\max} = \frac{\pi v}{a}$$

  - Relative error of considering an finite integration region:

$$\left| \frac{I_1 - I_2}{I_2} \right|$$

I_1 Integration up to $|\omega| < +\infty$

I_2 Integrating up to $|\omega| < \omega_{\max}$



$a)\ a = 100\ nm \quad b)\ a = 250\ nm \quad c)\ a = 700\ nm$

  - For a=100 nm the error is much less than 10% in most of the cases.

Field Characterization

- Imposing the classical boundary conditions (continuity of H_z and E_x):

$$\lfloor H_z \rfloor_{y=0} = 0 \qquad \left\lfloor \frac{1}{\varepsilon}\frac{\partial H_z}{\partial y} \right\rfloor_{y=0} = 0$$

The scattering problem is underdetermined!

- We need 2 additional boundary conditions to solve the problem: **the microscopic electric current and the additional potential must be continuous at y=0:**

M. G. Silveirinha, *IEEE Transactions on Antennas and Propagation*, vol.54, no.6, pp. 1766- 1780 (2006).

$$\lfloor I_y \rfloor_{y=0} = 0 \qquad I_y = a^2 \frac{-j\mathbf{k}_t}{k_t^2} \cdot \left[j\omega\varepsilon_0\varepsilon_h \frac{\partial \mathbf{E}_t}{\partial y} + \left(\frac{\omega^2}{c^2}\varepsilon_h - k_t^2 \right) \left(\hat{\mathbf{y}} \times \mathbf{H}_t \right) \right]$$

$$\lfloor \varphi \rfloor_{y=0} = 0 \qquad \varphi = -\frac{a^2}{j\omega C} \frac{\omega\varepsilon_0\varepsilon_h \mathbf{k}_t}{k_t^2} \cdot \left[\frac{\partial^2 \mathbf{E}_t}{\partial y^2} + \left(\frac{\omega^2}{c^2}\varepsilon_h - k_t^2 \right) \mathbf{E}_t \right]$$