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On-Net and Off-Net Pricing On Asymmetric Telecommunications Networks*

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Abstract

The differential between on-net and off-net prices, for example on mobile telephony networks, is an issue that is hotly debated between telecoms operators and regulators. Small operators contend that their competitors' high off-net prices are anticompetitive. We show that if the utility of receiving calls is taken into account, the equilibrium pricing structures will indeed depend on firms' market shares. Larger firms will charge higher off-net prices even without anticompetitive intent, both under linear and two-part tariffs. Predatory behavior would be accompanied by even larger on-net / off-net differentials even if access charges are set at cost.

Keywords: Telecommunications network competition, on/off-net pricing, asymmetry, call externality

JEL: L51

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1 Introduction

This is not a paper about access charges (or termination charges, as they are called in regulatory practice). It is centered instead on the setting of *retail prices* when networks price discriminate between on-net and off-net calls. While this problem has already been touched upon several times in the literature, we take the additional steps of allowing for call externalities and for asymmetric networks, both of which have significant effects on the market outcome.

The markets that we are trying to model are the retail markets of mobile telephony in the European Union. Retail prices are not regulated, only the caller pays for the call (if he is not roaming), price discrimination between on-net and off-net calls is the rule rather than the exception, both linear and two-part tariffs are on offer, market shares of networks vary widely, and consumers care about being called. The "tariff-mediated network externality" created by the on-net / off-net price differential has led small networks to complain that it puts them at a disadvantage, or even that large networks can use this price differential strategically to induce their exit.

In the following we will analyze Nash equilibria in the presence of price discrimination between on-net and off-net calls, and also "predatory pricing", where the large network tries to leverage the tariff-mediated network externality to reduce the small network's profits. This is done for both linear and two-part tariffs. We will check whether on the one hand predation is successful, and on the other whether it is detectable (distinguishable from the Nash equilibrium). In other words, we will provide some evidence as to whether the claim of "predatory on-net / off-net price discrimination" can make sense or not.

So what about access charges? Since here we are not interested in the question of anti-competitive or collusive access charges we assume for simplicity that they are set by an industry regulator. This assumption corresponds to regulatory reality in the European Union, where following the introduction of the telecommunications Directives of 2002 regulators must implement price controls for access charges on all mobile networks.

Our results are as follows: We find that both asymmetry and the call externality have strong effects on the equilibrium on-net and off-net prices, and the resulting on-net / off-net differentials: Large firms charge significantly higher off-net prices, and create a higher on-net / off-net differential. As a result, even with a balanced calling pattern, where each consumer calls every other consumer with the same probability, the traffic between the two networks will not be balanced: The small network incurs a permanent access deficit if reciprocal access charges are set above cost. This is true under both linear and two-part tariffs. Furthermore, under linear tariffs the large network also charges a higher on-net price, while with two-part tariffs both firms set the on-net price at the efficient level.

We present a series of comparative statics results, with respect to the level of asymmetry in market share, the size of the call externality, product differentiation and a reciprocal access charge. These are derived analytically for small asymmetries, and contrasted with results from numerical simulations for larger asymmetries.

As concerns predatory pricing, we find that its hallmark is a large on-net / off-net differential. With both types of tariffs, the predating firm's off-net price is increased above the Nash equilibrium level in order to reduce the small firm's access revenue (if access is priced above cost), and to reduce the call externality enjoyed by the small firm's customers. On the other hand, the distinction between the predatory and Nash equilibrium scenarios is not easy in practice. The difference between the two is quantitative rather than qualitative, and regulators or competition authorities very likely do not possess the necessary information to make an informed judgement.

Section 2 contains a short overview of the literature, and Section 3 introduces the model. Section 4 considers the profit-maximizing on-net / off-net pricing structures and the Nash equilibria of the pricing game. Section 5 discusses anti-competitive behavior by the large firm, while Section 6 concludes.

2 Overview of the Literature

Work published in this area has considered several of the aspects and assumptions central to this paper. The seminal paper in the literature on price discrimination between on-net and off-net calls is Laffont, Rey and Tirole (1998, LRT), which was later followed by Gans and King (2001). Both papers only consider symmetric equilibria, which leads to simple and elegant expressions for equilibrium values.

There is a budding literature on competition between asymmetric networks. Carter and Wright (1999, 2003) introduce asymmetry through an additive component in consumers' utility function. Cambini and Valletti (2004) endogenize the value of this parameter in a game of quality choice by networks. Yet, these articles do not consider tariff-mediated price discrimination.

De Bijl and Peitz (2002, ch. 6.4) present the equilibrium pricing structure with two-part tariffs and tariff-mediated price discrimination, but in the absence of a call externality. In this case both the on-net and off-net prices are equal to cost, and therefore the differential is completely determined by the access charge. As we will see below, if the call externality is taken into account then strategic considerations change this result. Dewenter and Haucap (2005) consider asymmetric networks and the setting of access charges when consumers are not aware of their level (comparable to what happens under roaming), but do not consider on-net calls.

The model of call externality used in the following has been introduced by Kim and Lim (2001) and Jeon, Laffont and Tirole (2004, JLT). While both papers are mainly concerned with the "receiver pays principle", JLT consider on pp. 104-105 the equilibrium pricing structure in two-part tariffs with asymmetric networks, and show that on-net and off-net prices can differ significantly from the underlying cost level. They do not solve for the equilibrium market shares, and therefore do not consider the equilibrium differential. Berger (2004) and Berger (2005) reconsider the role of reciprocal access charges in the presence of a call externality, with linear or two-part tariffs, respectively.

Lastly, to our knowledge there is no analysis of the on-net / off-net differential in the presence of predatory pricing. This is true even at the basic level of analysis presented below, where we only consider what some limited form of predatory pricing would look like, and not whether predation as such is rational. In particular, the question we tackle here is fundamentally different from foreclosure through high access charges, see e.g. Gabrielsen and Vagstad (2004) or Calzada and Valletti (2005).

3 The Model

The following model joins elements from LRT, Carter and Wright (1999) and JLT. Two telecommunications networks are situated at the extreme points of a Hotelling line, with firm 1 at point 0, and firm 2 at point 1. Each network supports a fixed cost per client of f_i and has constant marginal costs of origination and transport of c_{0i}, and of termination of c_{ti}, with resulting on-net cost $c_i = c_{0i} + c_{ti}$. Network i receives an access charge of a_i for terminating calls from its competitor, resulting in off-net costs $c_{fi} = c_{0i} + a_j$. In order to concentrate on the setting of retail prices we assume that access charges are set by a regulator. An example of the situation portrayed here is that of two competing mobile networks whose access charges are regulated. The latter now is usual in the EU, following the 2002 set of directives on telecommunications. Denote the market share of network i by α_i, with $\alpha_1 + \alpha_2 = 1$ since we assume that the whole market is covered in equilibrium.

Firms set either linear prices or two-part tariffs, and price discriminate between on-net and off-net calls. Network i's prices for on-net and off-net calls, and the fixed fee, are p_{ii}, p_{ij} and F_i, respectively, with $i, j \in \{1, 2\}$, $j \neq i$. For a linear tariff we simply set $F_i = 0$ in the following expressions.

A mass 1 of consumers is distributed uniformly along the Hotelling line. The consumer at location x has a utility loss of $\frac{1}{2\sigma}|x - l|$ if he subscribes to the network at location l. Furthermore, similar to Carter and Wright (1999), consumers receive an additional utility $\beta = A/\sigma$ if they join network 1, where A is the *ex ante* asymmetry in market share (before equilibrium effects). This assumption models an incumbency or reputation advantage of network 1. Its purpose is to make the market equilibrium asymmetric, with $\alpha_1 > \alpha_2$.

As in JLT consumers receive utility by making and receiving calls. The direct utility of making calls is $u(q)$, where q is the length of the call in minutes, and if the price per minute is p, the indirect utility is $v(p) = \max_q \{u(q) - pq\}$. The associated demand function is $q_{ij} = q(p_{ij})$. In the following we will use a constant elasticity demand function $q(p) = p^{-\eta}$, where $\eta > 1$, thus $u(q) = \frac{\eta}{\eta-1} q^{\frac{\eta-1}{\eta}}$ and $v(p) = \frac{1}{\eta-1} p^{1-\eta}$. The utility of receiving a call of duration q is $\gamma u(q)$, where $\gamma \in [0, 1]$.

For simplicity we assume a balanced calling pattern, i.e. each consumer calls each other consumer with the same probability, independent of which network they belong to. This does *not* imply that the actual traffic will be balanced, because the lengths of calls depend on their respective prices per minute (which will differ in equilibrium).

The utilities of subscribing to network 1 or 2 are

$$U_1(x) = w_1 + \beta - \frac{1}{2\sigma}x, \; U_2(x) = w_2 - \frac{1}{2\sigma}(1 - x) \tag{1}$$

where

$$\begin{aligned} w_i &= \alpha_i [v(p_{ii}) + \gamma u(q_{ii})] + \alpha_j [v(p_{ij}) + \gamma u(q_{ji})] - F_i & (2) \\ &= \alpha_i h_{ii} + \alpha_j h_{ij} - F_i & (3) \end{aligned}$$

where $h_{ij} = v(p_{ij}) + \gamma u(q_{ji})$. The indifferent consumer is located at $x = \alpha_1$, therefore

$$\alpha_1 = \frac{1}{2} + A + \sigma(w_1 - w_2). \tag{4}$$

This implicit equation for α_1 can be solved for

$$\alpha_1 = \frac{1/2 + A + \sigma(h_{12} - h_{22} - F_1 + F_2)}{1 + \sigma(h_{12} + h_{21} - h_{11} - h_{22})} = \frac{H_1}{H}. \tag{5}$$

Firms' profits are described by the standard expression

$$\pi_i = \alpha_i \left[\alpha_i \left(p_{ii} - c_i\right) q_{ii} + \alpha_j \left(p_{ij} - c_{fi}\right) q_{ij} + F_i - f_i + \alpha_j \left(a_i - c_{ti}\right) q_{ji}\right] \quad (6)$$

Consumer surplus is given by

$$CS = \int_0^{\alpha_1} U_1(x)\,dx + \int_{\alpha_1}^1 U_2(x)\,dx = \alpha_1 \left(w_1 + \frac{A}{\sigma}\right) + \alpha_2 w_2 - \frac{\alpha_1^2 + \alpha_2^2}{4\sigma}. \quad (7)$$

Total welfare is $W = CS + \pi_1 + \pi_2$, which can be written as

$$\begin{aligned} W &= \alpha_1^2 \left[(1+\gamma) u(q_{11}) - c_1 q_{11}\right] + \alpha_2^2 \left[(1+\gamma) u(q_{22}) - c_2 q_{22}\right] \\ &\quad + \alpha_1 \alpha_2 \left[(1+\gamma)\left(u(q_{12}) + u(q_{21})\right) - c_1 q_{12} - c_2 q_{21}\right] \\ &\quad + \alpha_1 \left(\frac{A}{\sigma} - f_1\right) - \alpha_2 f_2 - \frac{\alpha_1^2 + \alpha_2^2}{4\sigma}. \end{aligned} \quad (8)$$

In particular, access profits cancel out. This expression indicates the known result that, for any fixed market shares α_1 and α_2 the socially optimal prices p_{ij} are all equal to $p_{ij}^{so} = c_i / (1+\gamma)$. These prices are below cost because they internalize the call externality. If $V_i^{so} = (1+\gamma) u(q_i^{so}) - c_i q_i^{so}$ is the welfare derived from the socially optimal number of calls between network i and both networks i or j, the socially optimal market share can be found by maximizing

$$\max_{\alpha_1} \alpha_1 \left(V_1^{so} + \frac{A}{\sigma} - f_1\right) + (1-\alpha_1)\left(V_2^{so} - f_2\right) - \frac{\alpha_1^2 + (1-\alpha_1)^2}{4\sigma}, \quad (9)$$

with solution

$$\alpha_1^{so} = \frac{1}{2} + A + \sigma\left(V_1^{so} - V_2^{so} - f_1 + f_2\right). \quad (10)$$

In particular, if firms' costs are identical, then the socially optimal market share of the large firm is $\alpha_1^{so} = \frac{1}{2} + A$.

4 The Equilibrium On-/Off-Net Pricing Structure

4.1 Linear tariffs

The main aim of this section is to characterize how the equilibrium on-net / off-net pricing structure depends on the asymmetry in market shares. We do not consider possibly anti-competitive conduct here; this will be done in Section 5.

First we consider linear prices, i.e. $F_i = 0$. Defining the Lerner indices $L_{ii} = (p_{ii} - c_i)/p_{ii}$ and $L_{ij} = (p_{ij} - c_{fi})/p_{ij}$, we obtain the following result:

Lemma 1 *For any given market share α_i, the on-net / off-net pricing structure of network i is characterized by the following relation between Lerner indices:*

$$L_{ij} = \frac{1}{\eta} + \frac{(1+\gamma\eta)^{-1} - \alpha_i}{1-\alpha_i}\left(L_{ii} - \frac{1}{\eta}\right). \tag{11}$$

The slope decreases in $\gamma\eta$ and α_i.

Proof. Using the identities $\frac{\partial\alpha_i}{\partial p_{ii}} = -\frac{\sigma\alpha_i q_{ii}(1+\gamma\eta)}{H}$ and $\frac{\partial\alpha_i}{\partial p_{ij}} = -\frac{\sigma q_{ij}(\alpha_j - \alpha_i\gamma\eta)}{H}$, the first-order conditions $\frac{\partial\pi_i}{\partial p_{ii}} = 0$ and $\frac{\partial\pi_i}{\partial p_{ij}} = 0$ can be written as

$$2\alpha_i R_{ii} + (1-2\alpha_i)(R_{ij} + Q_i) - \alpha_i \frac{H(1-\eta L_{ii})}{\sigma(1+\gamma\eta)} = f_i, \tag{12}$$

$$2\alpha_i R_{ii} + (1-2\alpha_i)(R_{ij} + Q_i) - \alpha_i \frac{\alpha_j H(1-\eta L_{ij})}{\sigma(\alpha_j - \alpha_i\gamma\eta)} = f_i, \tag{13}$$

with $R_{ii} = (p_{ii} - c_i)q_{ii}$, $R_{ij} = (p_{ij} - c_{fi})q_{ij}$ and $Q_i = (a_i - c_{ti})q_{ji}$. Equating the two leads to

$$(\alpha_j - \alpha_i\gamma\eta)(1-\eta L_{ii}) - \alpha_j(1+\gamma\eta)(1-\eta L_{ij}) = 0, \tag{14}$$

which can be solved for L_{ij}. ■

This relation between both Lerner indices is a straight line which passes through the monopoly point $L_{ii} = L_{ij} = \frac{1}{\eta}$. If there is no call externality then both on-net and off-net Lerner indices are equal, as in LRT. On the other hand, if γ is positive but small ($\gamma < \frac{\alpha_j}{\eta\alpha_i}$ or $\alpha_i < \frac{1}{1+\gamma\eta}$) then L_{ij} increases with L_{ii}, while L_{ij} may even decrease (from above towards the monopoly value) if γ is large ($\gamma > \frac{\alpha_j}{\eta\alpha_i}$), as Berger (2004) has shown in the symmetric case. Still, we always have $L_{ij} \geq L_{ii}$, which implies that if access is priced at or above cost then off-net prices are always higher than on-net prices.

The off-net Lerner indices are higher than the corresponding on-net ones because the call externality confers additional utility to the clients of the rival network. By raising its off-net price a network will limit the number of call minutes that reach these clients, and therefore improve its relative competitive position. Note that naturally a higher off-net price of network i

as such reduces its attractiveness, but this is no contradiction to the above argument since clients of *both* networks are made worse off. If we start out from the equilibrium off-net prices in the absence of a call externality, then taking this externality into consideration provides an incentive to raise the off-net price above its previous equilibrium value. In mathematical terms, the direct effect of a higher off-net price on own profits is a second-order effect (since p_{ij} has been chosen optimally), while the indirect effect caused by the call externality is of first order.

Two important observations follow:

1. Since the slope of the relationship in (11) decreases in α_i, the firm with the larger market share would have a higher off-net Lerner index if on-net Lerner indices were equal. That is, for similar off-net costs (including access charges) and on-net prices, the large network's off-net price will be higher than the small network's. This effect would only be reversed if the small network were to choose significantly higher on-net prices (We will see below that this does not happen in equilibrium).

2. Since the off-net price is lower in the small network if on-net prices are similar, there will be an imbalance in interconnected traffic between both networks even under a balanced calling pattern, with an access deficit persistently affecting the profits of the small network. This deficit results from the internalization of the call externality of on-net receivers, which is stronger on the larger network. Therefore it does not result from anti-competitive behavior.[1][2]

The Nash equilibria in linear tariffs are characterized by conditions (12) and (14) for firm $i = 1, 2$. As shown in the previous literature, these equilibria will exist if σ and γ are close enough to zero and a_i close enough to c_{ti}. Symmetric equilibria have been characterized by Berger (2004) using a graphical method, since they cannot be determined analytically. In the asymmetric case not even a graphical method is feasible since four prices (instead of only two) are involved, therefore equilibrium values can only be found through numerical solutions.[3]

The comparative statics of equilibrium under small asymmetries are qual-

[1] Asymmetric access prices, with the large network charging less per minute, could alleviate this problem, see Peitz (2005).

[2] Later in Section 5 we will see that there can be an anti-competitive role for increasing off-net prices even further.

[3] The algorithms have been implemented in Matlab 6.5, and are available from the author on request. For all numerical results presented in the following we have taken care to check that the second-order and boundary conditions are satisfied, and that the equilibria are stable.

itatively identical to the symmetric case, due to continuity (unless a derivative in the symmetric case is zero). Therefore we first state the comparative statics for the symmetric case, where both networks have the same size, and then contrast these with numerical results if we find differences for larger asymmetries. In point 1 below we show how equilibrium prices change if a small asymmetry is introduced., while in points 2 and 3 we offer a new analytical proof for the results of Lemma 1 in Berger (2004), and add the corresponding results about changes in the on/off-net differential.

Proposition 2 *Starting from a symmetric Nash equilibrium with linear tariffs, the following comparative statics results hold:*

1. *On- and off-net prices, and the on/off-net differential, of the large (small) network increase (decrease) with the introduction of a small ex ante asymmetry A, if $\gamma\eta < k$ for some $k > 1$, per customer fixed cost f_1 are small enough, and $a \geq c_{ti}$.*

2. *On-net prices decrease with reciprocal access charge a, while off-net prices increase. Therefore the on/off-net differential is increasing in a.*

3. *On-net prices decrease with the intensity of competition σ, while off-net prices decrease (increase) if $\gamma\eta < (>) 1$. The on/off-net differential is decreasing (increasing) if γ is close to zero and $a > c_{ti}$ (γ large enough or $a < c_{ti}$).*

4. *On-net prices decrease with the call externality γ, while off-net prices and on/off-net differentials increase if either $\gamma\eta \geq 1$, or if $\gamma\eta < 1$ and $a - c_{ti}$ is small enough. They may decrease if $\gamma\eta < 1$ and $a - c_{ti}$ is large enough.*

Proof: See appendix.

Figure 1 illustrates how asymmetry of network size affects the equilibrium. Prices for the small (large) firm correspond to equilibrium market shares smaller (larger) than one half. Apart from setting lower prices, the small firm chooses a significantly smaller on-net / off-net differential. Since the larger firm charges higher prices, its equilibrium market share will be smaller than $\frac{1}{2} + A$, while its profits will be higher than at the symmetric equilibrium with equal market shares.

<< Figure 1 >>

The comparative statics in Proposition 2 hold for a small asymmetry. Some results are reversed for large asymmetries. In numerical simulations we found the following differences in the behavior of large and small firms, see Hoernig (2006):

1. Call externality γ: The cut-off value for γ above which off-net price decreases in γ is lower for the smaller firm, i.e. the small firm's off-net price may be decreasing in γ while the large firm's still increases.

2. Reciprocal access charge a: The small network's on-net price decreases faster with the access charge than the large network's on-net price. The large network's off-net price may increase faster than the access charge, while the small network's increases slower.

4.2 Two-part tariffs

Jeon, Laffont and Tirole (2004, p. 105) and Berger (2005) derive the profit-maximizing pricing structure. Keeping market share α_i constant, they substitute

$$F_i = \alpha_i h_{ii} + \alpha_j v(p_{ij}) - \alpha_i \gamma u(q_{ij}) + K_i \tag{15}$$

into π_i, where K_i does not depend on p_{ii} or p_{ij}. Maximizing π_i with respect to these variables then leads to

$$p_{ii} = \frac{c_i}{1+\gamma},\ p_{ij} = \frac{c_{fi}}{1-\gamma\alpha_i/\alpha_j} \text{ if } \alpha_i < \frac{1}{1+\gamma},\ p_{ij} = \infty \text{ otherwise.} \tag{16}$$

In terms of Lerner indices,

$$L_{ii} = -\gamma, L_{ij} = \frac{\alpha_i}{\alpha_j}\gamma \text{ if } \alpha_i < \frac{1}{1+\gamma},\ L_{ij} = 1 \text{ otherwise.} \tag{17}$$

On-net prices internalize receivers' utility of receiving calls, leading to the efficient price below marginal cost. On the other hand, off-net prices remain above marginal cost and increase in own market share (towards infinity as α_i approaches $1/(1+\gamma)$, while the Nash equilibrium still exists). Again, the higher off-net price reduces the rival network's attractiveness through limiting the number of call minutes its customers will receive.

Last but not least, the equilibrium fixed fee is

$$F_i = f_i + \alpha_i \frac{H}{\sigma} - 2\alpha_i R_{ii} + (\alpha_i - \alpha_j)(R_{ij} + Q_i). \tag{18}$$

It increases in α_i at $\alpha_i = \frac{1}{2}$, therefore at least for similar market shares the fixed fee is larger for the large firm.

We now derive some comparative statics results for two-part tariffs, where for given competitor's prices (p_{jj}, p_{ji}, F_j) firm i solves $\max_{p_{ii}, p_{ij}, F_i} \pi_i$. Since also in this case the equilibrium market share cannot be found analytically, we present again the comparative statics in symmetric equilibrium.

Proposition 3 *Starting from a symmetric Nash equilibrium with two-part tariffs, the following comparative statics results hold:*

1. *Fixed fee, off-net prices and the on/off-net differential of the large (small) network increase (decrease) with a small ex ante asymmetry A. On-net prices do not change.*

2. *Off-net prices and the on/off-net differential increase, and fixed fees decrease, with reciprocal access charge a. On-net prices do not change.*

3. *Both on- and off-net prices do not change with the intensity of competition σ (the latter result is not robust to small asymmetries), but the fixed fees decrease.*

4. *On-net prices decrease with the call externality γ, while off-net prices and on/off-net differentials increase (for $\alpha_1 < \frac{1}{1+\gamma}$). The fixed fee decreases if $a \geq c_t$.*

Proof. Together with equations (16) and (18), the condition

$$T \equiv \alpha_1 - \frac{1}{2} - A + \sigma (w_1 - w_2) = 0$$

describes the equilibrium market share. For σ small enough $\partial T / \partial \alpha_1 > 0$, thus $d\alpha_1 / dA > 0$. With equal costs and at $A = 0$ (and thus at $\alpha_1 = \frac{1}{2}$),

$$\frac{dF_i}{d\alpha_i} = \frac{H}{\sigma} + 2 \left((p_{ij} - c_i) q_{ij} - (p_{ii} - c_i) q_{ii} \right) > 0,$$

and $dp_{ij} / d\alpha_i > 0$. The other results follow from $p_{ii} = \frac{c_i}{1+\gamma}$, $p_{ij} = \frac{c_{fi}}{1-\gamma}$ and

$$F_i = f_i + \frac{1}{2\sigma} + \frac{1 + \gamma\eta}{\eta - 1} \left(\frac{c_{fi}}{1 - \gamma} \right)^{1-\eta} - \frac{\gamma + 1}{\eta - 1} \left(\frac{c_i}{1 + \gamma} \right)^{1-\eta}$$

at a symmetric equilibrium. In particular,

$$\frac{dF_i}{d\gamma} = \frac{1 - \gamma\eta^2}{1 - \gamma} v (p_{ij}) - \eta v (p_{ii}),$$

which is negative if $a \geq c_t$. ■

As with linear tariffs, the large firm's profits will be higher, and the small firm has an access deficit due to its lower off-net price. Comparative statics results that arise with larger asymmetries are:

1. Call externality γ: the larger firm's off-net price rises faster than the small firm's.
2. Intensity of competition σ: With a larger σ, the large firm's off-net price may go up while the small firm's decreases. Even so, these changes are small, and the main effect is the reduction in the fixed fee.
3. Reciprocal access charge a: The larger firm's off-net price and differential increase faster with the access charge than the small firm's.

Thus under both linear and two-part tariffs we find the result that larger firms charge higher equilibrium off-net prices and that their pricing decisions result in larger on/off-net differentials. Higher reciprocal access charges widen this differential even further.

5 Can Termination-based Price Discrimination Be Used Anti-competitively?

Until now we have assumed that both firms try to maximize their profits, which results in the standard Nash equilibrium of the game. A completely different question is that of anti-competitive or predatory pricing, in which firm 1 tries to make firm 2 leave the market, or hinder its normal development, by targeting its profits, more specifically by minimizing them.

This can obviously be done by choosing arbitrarily low on-net prices p_{11}, driving the market share and profits of firm 2 to zero. At the same time, high off-net prices can reduce the utility that clients of the other network obtain by receiving calls. Therefore the *possibility* of predation is easily established if we are willing to let firm 1 inflict arbitrary losses on itself (in the short run).

A more interesting question is the following: What is the on-net / off-net pricing structure that emerges from "limited" predation? Instead of provoking immediate exit, the large firm may restrict the small firm's profits and cash flows, which makes it more difficult for this firm to invest either in customer retention or improvement of the network. The aspects that we consider in this framework are: For a given target profit level of firm 2, how does firm 1 trade off optimally between low on-net and high off-net prices? Can this pricing structure be distinguished, especially under limited information about costs and demand, from Nash equilibrium pricing?

It is worth noting that the notion of predation that we adopt here is not the "Areeda-Turner standard" invoked in the antitrust jurisdiction of the USA, that is, the setting of prices below some measure of cost. This standard has been criticized as being inconsistent with economic theory. Rather, we follow the definition of Ordover and Willig (1981), with predation a "response to a rival that sacrifices part of the profit that could be earned under competitive circumstances, were the rival to remain viable, in order to induce exit and gain consequent additional monopoly profit" (pp.9-10). This definition can be extended to one of "softening-up of the victim" see e.g. Tirole (1988, ch. 9) and Church and Ware (2000, ch. 21). In this context we are not interested in the dynamic aspects (future entry), nor in the rationality of predatory behavior. The simple question that we pose is what the market would look like in the short run in the presence of predatory behavior.

We consider the following predation game, whose Nash equilibrium we will call the "predation equilibrium": With linear tariffs firm 1 solves, given (p_{22}, p_{21}), and some maximum profit level $\bar{\pi}_2$ of firm 2,

$$\max_{p_{11},p_{12}} \pi_1 \; s.t. \; \pi_2 \leq \bar{\pi}_2. \tag{19}$$

Firm 2 maximizes π_2 over (p_{22}, p_{21}), given (p_{11}, p_{12}), just as before. By lowering $\bar{\pi}_2$ towards zero (or below zero) we can reproduce "unconstrained" predation.

As a first step we consider firm 1's optimal pricing structure for fixed market shares. We find the following:

Lemma 4 *In the predation equilibrium with linear tariffs, the on-net / off-net pricing structure of the predating firm* 1 *is characterized by the following relation:*

$$L_{12} = \frac{1}{\eta} + \frac{(1+\gamma\eta)^{-1} - \alpha_1}{1-\alpha_1}\left(L_{11} - \frac{1}{\eta}\right) + \mu\frac{a_2 - c_{t2}}{p_{12}} \tag{20}$$

where $\mu \geq 0$ *is the Lagrange multiplier of the condition* $\pi_2 \leq \bar{\pi}_2$. *The predated firm 2's pricing structure is given by (11).*

Proof. As in the derivation of Lemma 1 we fix market share α_1, and solve, given (p_{22}, p_{21}),

$$\max_{p_{11},p_{12}} \pi_1 \; s.t. \; \alpha_1 - \frac{1}{2} - A - \sigma(w_1 - w_2) = 0, \; \pi_2 \leq \bar{\pi}_2.$$

With Lagrange multipliers λ and μ, first-order conditions are

$$\begin{aligned} \alpha_1\left(1-\frac{p_{11}-c_1}{p_{11}}\eta\right)+\lambda\sigma\left(1+\gamma\eta\right) &= 0 \quad (21) \\ \alpha_1\alpha_2\left(1-\frac{p_{12}-c_{1f}}{p_{12}}\eta\right)+\lambda\sigma\left(\alpha_2-\alpha_1\gamma\eta\right)+\mu\alpha_1\alpha_2\frac{a_2-c_{t2}}{p_{12}}\eta &= 0. \end{aligned}$$

Substituting out λ and solving for L_{12} we obtain the above result. As concerns firm 2 nothing changes since it solves the same problem as before. ■

This result means that under predation the relation describing firm 1's pricing structure shifts if access is not priced at cost: With an access charge above cost, firm 1's off-net price will be even higher. This is caused by the positive effect of terminating calls from network 1 on firm 2' profits: If the small firm's access price is above cost then the large firm, by further increasing its off-net price, restricts the off-net minutes terminated on the small network in order to reduce its termination revenues. In the theoretical case of access charges below cost firm 1 would adopt the opposite strategy: Since firm 2 would lose money on every call it receives, the large network would choose a lower off-net price to increase the number of these calls.

On the other hand, if access is priced at cost then the relation between the two prices does not change. Nevertheless, market shares and the overall price levels will differ in predation equilibrium, therefore in any case we now must consider the full equilibrium as in Figure 2. For simplicity we only consider cost-based access, therefore the additional effect just identified is absent.

<< Figure 2 >>

In this numerical example the most right-hand value of firm 2's profits corresponds to the Nash equilibrium profits (without predation, that is). At this profit level the predation equilibrium prices coincide with the prices in the Nash equilibrium. This is intuitive since, given firm 2's Nash equilibrium prices, firm 1 can only obtain its Nash equilibrium profits by choosing its Nash equilibrium prices. Any intensification of predation corresponds to a leftward movement to a lower profit level of firm 2.

We find the following:

Remark 5 *If firms compete in linear tariffs, and if the large firm practices "limited predation", then*

1. *As the degree of predation increases:*

(a) *The large firm's on-net price falls rapidly, while its off-net price first decreases and then increases above the Nash equilibrium level. As a consequence, the large firm's on-net / off-net differential increases strongly.*

(b) *The small firm's on-net and off-net prices both decrease slowly, leading to a slight reduction in the on-net / off-net differential.*

2. *There are "decreasing returns to scale" in predation: Any further reduction in the small firm's profit is bought at increasing cost for the large firm.*

The large firm decreases its on-net price strongly to steal customers from the small network. On the other hand, the setting of off-net prices results from two opposing incentives: A lower off-net price attracts consumers from the other network, while a higher off-net price has the two functions of restricting the call externality on the small network and of reducing its termination revenues. As the market share of the small network becomes smaller with an increasing level of predation, the off-net price becomes less important for consumers on the large network, while receiving calls from the large network becomes essential for consumers on the small network. Therefore it is optimal for the large network to increase its off-net price significantly, even if access is priced at cost as in this example.

With two-part pricing, firm 2 responds as above in (16), while firm 1 now solves the predation problem with a two-part tariff. The equilibrium pricing structure is described in the following Lemma:

Lemma 6 *In the predation equilibrium with two-part tariffs, the on-net / off-net pricing structure of the predating firm* 1 *is characterized by the following relations:*

$$L_{11} = -\gamma,\ L_{12} = \frac{\alpha_1}{\alpha_2}\gamma + \mu\frac{a_2 - c_{t2}}{p_{12}} \text{ if } \alpha_1 < \frac{1}{1+\gamma},\ p_{12} = \infty \text{ otherwise.} \quad (22)$$

where $\mu > 0$ *is the Lagrange multiplier of the condition* $\pi_2 \leq \bar{\pi}_2$. *The predated firm 2's pricing structure is given by (16).*

Proof. First substitute F_1 as in (15) into profits π_1. Again keeping market shares constant, firm 1's optimal pricing structure solves, given (F_2, p_{22}, p_{21}),

$$\max_{p_{11}, p_{12}} \pi_1 \ s.t.\ \pi_2 \leq \bar{\pi}_2.$$

While the first-order condition for p_{11} does not change, for p_{12} it becomes

$$p_{12} : 0 = \left(1 - \frac{(p_{12} - c_{1f})}{p_{12}}\eta\right) - \left(1 - \frac{\alpha_1}{\alpha_2}\gamma\eta\right) + \mu\frac{a_2 - c_{t2}}{p_{12}}\eta = 0. \qquad (23)$$

Solving these equations for p_{11} and p_{12} leads to the above results. As firm 2 solves the same problem as before, its pricing structure does not change. ■

As compared to (16) the on-net price maintains its (efficient) value, but the off-net price increases if the access charge exceeds cost. That is, we encounter the same effect as with linear tariffs: The off-net priced is changed to reduce access profits of the small network.

The level of the off-net price still depends on the unknown equilibrium market shares. This implies that we again need to solve numerically for the equilibrium in order to determine the resulting pricing structure. The results are presented in Figure 3, again starting from the right at the Nash equilibrium profit level (with cost-based access).

<< Figure 3 >>

Remark 7 *If firms compete in two-part tariffs, and if the large firm practices "limited predation", then*

1. *As the degree of predation increases (i.e. the small firm's profits decrease):*

   (a) *Both firms' on-net prices remain constant. The large firm's off-net price increases strongly, while the small firm's off-net price decreases weakly.*

   (b) *The large firm's on-net / off-net differential increases strongly, while the small firm's decreases slightly.*

   (c) *Both fixed fees decrease, with the large firm's eventually being smaller.*

2. *There are "decreasing returns to scale" in predation: Any further reduction in the small firm's profit is bought at increasing cost for the large firm.*

Again even limited predation has some effects. What distinguishes predation with two-part tariffs from the Nash equilibrium is a high off-net price and a low fixed fee by the large firm. Consumers are attracted to the large network through the lower fixed fee, while call externalities are restricted through higher off-net prices. Contrary to the case of linear tariffs, the on-net price does not move, since it was already set at the efficient (and therefore in this context profit-maximizing) level.

The main feature shared by predation under both linear and two-part tariffs is the fact that the on/off-net price differential of the large firm increases significantly, while that of the small firm does not change much. In both cases the off-net price is raised strongly in order to reduce the call externality enjoyed by the small firm's clients. The main competitive weapons, though, are the on-net price under linear tariffs and the fixed fee under two-part tariffs.

One may ask whether the presence of the call externality makes any difference. In fact, it is decisive for this outcome. In the absence of a call externality, with linear pricing the on-net/off-net differential is driven mainly by the access charge, even under predation. The presence of the call externality leads to significantly higher off-net prices by the predating firm, and therefore to a much larger differential. With two-part tariffs, both on-net and off-net prices are equal to cost if there is no call externality, even under predation. Therefore the differential is constant and only depends on the access charge. In the presence of the call externality, this differential is driven by the difference in market shares and strategic considerations.

Last but not least, we turn to the question of how a regulator or competition authority could distinguish between predatory and Nash equilibrium behavior. As we have seen above, the large firm's off-net price, and the on/off-net differential, will be larger under predation. There is no breakdown of communication, at least as long as the call externality is small enough. There is then a quantitative difference in behavior (which may be large, nevertheless), rather than a qualitative one. The distinction of the two types of behavior, if it is to be based on market data, could in principle be done by calibrating market equilibrium models. If the necessary information is not available then international comparisons may help at least to identify extreme cases.

6 Conclusions

We have presented a model where a large and a small telecommunications network compete in either linear or two-part tariffs. Our focus was on the differential between on- and off-net prices. We found that this differential is driven not only by the level of termination charges, but also by the utility of receiving calls (the call externality) and the relative size of networks.

In Nash equilibrium, the large network charges significantly higher off-net prices, and sets a higher on-net / off-net differential. This happens because the presence of the call externality gives incentives to the large network to limit off-net calls in order to make the smaller network less attractive. Our result is true under both linear and two-part tariffs, therefore it does not depend on the pricing structure.

In a second step we considered how the large network's pricing decisions would differ if it were to engage in some predatory activity against its smaller rival, i.e. if it were trying to hold down its profits. The off-net price and the differential increase, as compared to Nash equilibrium, for two reasons. First, the presence of a positive margin between access charge and access cost, at the smaller network, creates access revenue from incoming calls. Therefore the large network sets an even higher off-net price in order to limit the number of outgoing calls and consequently the small network's access revenue. In this respect, asymmetric access charges, i.e. higher charges at the smaller network, aggravate potential problems arising from on/off-net differentials at the large network.

Second, the large network competes more vigorously using lower on-net prices if competition is in linear tariffs, and lower fixed fees if competition is in two-part tariffs. This is usually accompanied by higher off-net prices. The resulting on-/off-net differential can be substantially larger than in Nash equilibrium. Thus even while a large differential may not be the main weapon for predation, it can indicate its presence.

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Appendix

Proof of Proposition 2:

Assume that firms are symmetric (identical costs and $A = 0$) and that access charges are reciprocal. At a symmetric equilibrium, the first-order conditions (12) and (14) can be stated as

$$\begin{aligned} S &= 2(1+\gamma\eta)(R_{11} - f_1) - \frac{H(1 - L_{11}\eta)}{\sigma} = 0, \\ T &= (1-\gamma\eta)(1 - L_{11}\eta) - (1+\gamma\eta)(1 - L_{12}\eta) = 0, \end{aligned}$$

where due to symmetry $H = 1 + 2\sigma(1+\gamma\eta)(v(p_{12}) - v(p_{11}))$. For any parameter $\theta \in \{a, \sigma, \gamma\}$ the comparative statics at a symmetric equilibrium are computed as

$$\begin{bmatrix} \frac{dp_{11}}{d\theta} \\ \frac{dp_{12}}{d\theta} \end{bmatrix} = -\begin{bmatrix} S_1 & S_2 \\ -t_1 & T_2 \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial S}{\partial \theta} \\ \frac{\partial T}{\partial \theta} \end{bmatrix} = \Delta \begin{bmatrix} S_2 \frac{\partial T}{\partial \theta} - T_2 \frac{\partial S}{\partial \theta} \\ -t_1 \frac{\partial S}{\partial \theta} - S_1 \frac{\partial T}{\partial \theta} \end{bmatrix},$$

where $\Delta = (S_1 T_2 + S_2 t_1)^{-1}$ is positive for $\gamma \leq \bar{\gamma}$ for some $\bar{\gamma} > \frac{1}{\eta}$, and we have defined

$$\begin{aligned} S_1 &= \frac{\partial S}{\partial p_{11}} = \frac{H\eta}{\sigma} \frac{c_1}{p_{11}^2} > 0, \\ S_2 &= \frac{\partial S}{\partial p_{12}} = 2q_{12}(1+\gamma\eta)(1 - L_{11}\eta) > 0, \\ t_1 &= -\frac{\partial T}{\partial p_{11}} = (1-\gamma\eta)\eta \frac{c_1}{p_{11}^2} > 0 \text{ if } \gamma\eta < 1, \\ T_2 &= \frac{\partial T}{\partial p_{12}} = (1+\gamma\eta)\eta \frac{c_{f1}}{p_{12}^2} > 0. \end{aligned}$$

We have given capital letters to positive terms and small letters to terms which become negative for $\gamma\eta > 1$, in order to sign the derivatives more easily. Moreover,

$$\begin{aligned} \frac{\partial S}{\partial a} &= 0, \; \frac{\partial T}{\partial a} = -\frac{\eta(1+\gamma\eta)}{p_{12}} < 0 \\ \frac{\partial S}{\partial \sigma} &= \frac{1 - L_{11}\eta}{\sigma^2} > 0, \; \frac{\partial T}{\partial \sigma} = 0 \\ \frac{\partial S}{\partial \gamma} &= \frac{\eta}{\sigma} \frac{1 - L_{11}\eta}{1+\gamma\eta} > 0, \; \frac{\partial T}{\partial \gamma} = -2\eta \frac{1 - L_{11}\eta}{1+\gamma\eta} = -2\sigma \frac{\partial S}{\partial \gamma} < 0 \end{aligned}$$

Thus

$$\begin{aligned}
\frac{\partial p_{11}}{\partial a} &= \Delta S_2 \frac{\partial T}{\partial a} < 0,\ \frac{\partial p_{12}}{\partial a} = -\Delta S_1 \frac{\partial T}{\partial a} > 0,\ \frac{\partial \left(p_{12} - p_{11}\right)}{\partial a} > 0. \\
\frac{\partial p_{11}}{\partial \sigma} &= -\Delta T_2 \frac{\partial S}{\partial \sigma} < 0,\ \frac{\partial p_{12}}{\partial \sigma} = -\Delta t_1 \frac{\partial S}{\partial \sigma} < 0 \text{ if } \gamma\eta < 1, \\
\frac{\partial \left(p_{12} - p_{11}\right)}{\partial \sigma} &= \Delta \frac{\partial S}{\partial \sigma} \left(T_2 - t_1\right)
\end{aligned}$$

The derivative of the on/off-net differential depends on the sign of $T_2 - t_1$. This is positive for $\gamma\eta > 1$, while it is negative at $\gamma = 0$ if $a > c_{ti}$, since in this case $L_{12} = L_{11}$, $p_{12} > p_{11}$, and

$$T_2 - t_1 = \frac{c_{f1}}{p_{12}^2} - \frac{c_1}{p_{11}^2} = \frac{1 - L_{12}}{p_{12}} - \frac{1 - L_{11}}{p_{11}} < 0.$$

As concerns the call externality γ, we have

$$\begin{aligned}
\frac{\partial p_{11}}{\partial \gamma} &= -\Delta \left(2\sigma S_2 + T_2\right) \frac{\partial S}{\partial \gamma} < 0,\ \frac{\partial p_{12}}{\partial \gamma} = \Delta \left(2\sigma S_1 - t_1\right) \frac{\partial S}{\partial \gamma}, \\
\frac{\partial \left(p_{12} - p_{11}\right)}{\partial \gamma} &= \Delta \left(2\sigma \left(S_1 + S_2\right) + T_2 - t_1\right) \frac{\partial S}{\partial \gamma}
\end{aligned}$$

Since $\left(2\sigma S_1 - t_1\right) = \left(2H - 1 + \gamma\eta\right) \eta c_1 p_{11}^{-2}$, if $\Delta > 0$ the off-net price increases in γ if and only if $2H - 1 + \gamma\eta > 0$. This is certainly true if $\gamma\eta > 1$ and the equilibrium is still stable ($H > 0$). On the other hand, this condition is equivalent to $1 + 4\sigma \left(v\left(p_{12}\right) - v\left(p_{11}\right)\right) > 0$. This latter condition is violated if p_{12} is large enough as compared to p_{11}, which can happen as a result of a large termination margin $a - c_{ti}$. That is, for $\gamma\eta < 1$ is if possible that p_{12} decreases in γ if the termination margin is large enough. Similar results hold for the on/off-net differential. The threshold termination margin after which this occurs is higher than for p_{12} because p_{11} always decreases in γ.

In the terms of the graphical representation of the symmetric equilibrium in Figure 1 of Berger (2004), the upward shift of curve (4) can be so strong that it supersedes the downward rotation of curve (3), and thus p_{12} decreases. The following numerical example proves that p_{12} may indeed be decreasing in γ in stable equilibrium: $c_i = 1$, $c_{ti} = 0.2$, $f_i = 0$, $\eta = 2$, $A = 0$, $\sigma = 0.8$. At $\gamma = 0$ both off-net price and differential are increasing if $a < 0.873$, while both are decreasing if $a > 1.346$; if a is between these values then the off-net price is decreasing, while the differential is still increasing.

As concerns the effect of asymmetry on prices, consider the following full system of equations describing the asymmetric Nash equilibrium, with

$G = 1 + \gamma\eta$:

$$\begin{aligned} 2\alpha_1 R_{11} + (1-2\alpha_1)(R_{12}+Q_1) - \alpha_1 \frac{H(1-\eta L_{11})}{\sigma G} &= f_1 \\ (1-G\alpha_1)(1-\eta L_{11}) - (1-\alpha_1) G (1-\eta L_{12}) &= 0 \\ 2(1-\alpha_1) R_{22} + (2\alpha_1 - 1)(R_{21}+Q_2) - (1-\alpha_1) \frac{H(1-\eta L_{22})}{\sigma G} &= f_2 \\ (1-G+G\alpha_1)(1-\eta L_{22}) - \alpha_1 G (1-\eta L_{21}) &= 0 \\ \frac{H_1}{H} - \alpha_1 &= 0 \end{aligned}$$

We have added a fifth equation for α_1 in order to keep derivatives simple. Denoting as above positive terms with capital letters, and terms that are positive only for $\gamma\eta < 1$ with small letters, we find that at $A = 0$ we have

$$\begin{bmatrix} \frac{\partial p_{11}}{\partial A} \\ \frac{\partial p_{12}}{\partial A} \\ \frac{\partial p_{22}}{\partial A} \\ \frac{\partial p_{21}}{\partial A} \\ \frac{\partial \alpha_1}{\partial A} \end{bmatrix} = - \begin{bmatrix} B & C & -D & C & -E \\ -j & K & 0 & 0 & -M \\ -D & C & B & C & E \\ 0 & 0 & -j & K & M \\ -N & -r & N & r & -1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{H} \end{bmatrix} = \frac{1}{\Phi} \begin{bmatrix} EK \\ Ej + M(B+D) \\ -EK \\ -(Ej + M(B+D)) \\ (B+D)K \end{bmatrix},$$

where $\Phi = H[2E(jr + KN) + (B+D)(K + 2Mr)] > 0$, and

$$\begin{aligned} B &= \frac{1}{2}\left(q_{11}(1-\eta L_{11}) + \frac{H\eta}{\sigma G}\frac{c_1}{p_{11}^2} \right),\ C = \frac{1}{2} q_{12}(1-\eta L_{11}),\ D = \frac{1}{2} q_{11}(1-\eta L_{11}), \\ E &= 2[(p_{12} - c_1) q_{11} - f_1],\ j = \left(1 - \frac{G}{2}\right)\eta \frac{c_1}{p_{11}^2},\ K = \frac{1}{2} G\eta \frac{c_{f1}}{p_{12}^2}, \\ M &= G\eta (L_{12} - L_{11}),\ N = \frac{\sigma G q_{11}}{2H},\ r = \frac{\sigma(2-G) q_{12}}{2H}. \end{aligned}$$

Thus p_{11}, p_{12} and α_1 are increasing in A, and p_{22} and p_{21} decreasing. This is true for $\gamma\eta < k$ for some $k > 1$, per customer fixed cost f_1 small enough, and $a \geq c_{ti}$. The change in the on/off-net differential is

$$\frac{\partial(p_{12} - p_{11})}{\partial A} = \frac{E(j-K) + M(B+D)}{\Phi}.$$

At $\gamma = 0$ we have : $j - K = \frac{1}{2}\eta \left(\frac{c_1}{p_{11}^2} - \frac{c_{f1}}{p_{12}^2} \right) = \frac{1}{2}\eta \left(\frac{1-L_{11}}{p_{11}} - \frac{1-L_{12}}{p_{12}} \right) \geq 0$ because $L_{12} \geq L_{11}$, and $p_{12} \geq p_{11}$ at least if $a \geq c_{ti}$. Thus the on-net differential increases with asymmetry as long as γ is small enough. ■

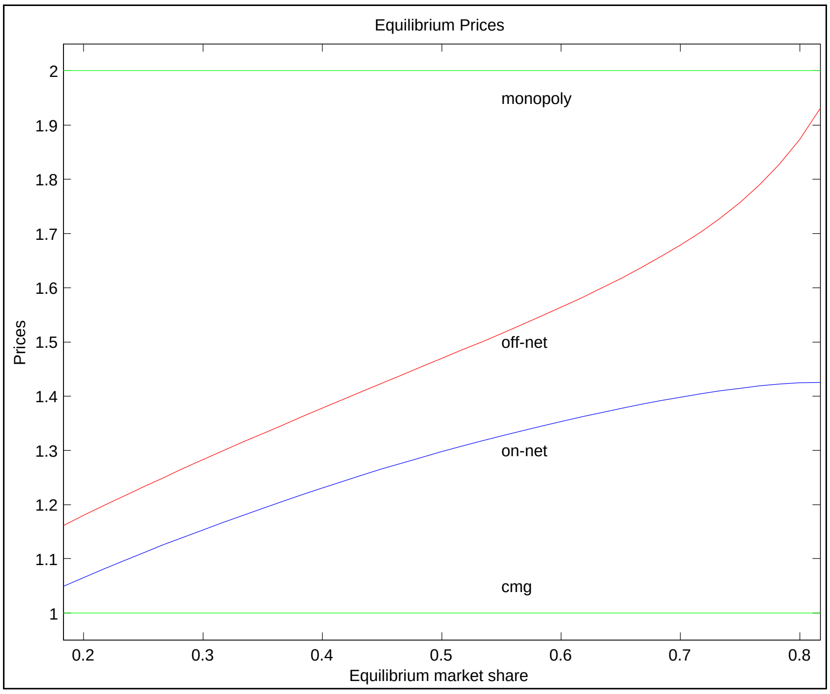

Figure 1: Equilibrium linear prices as A changes between -0.5 and 0.5. Parameter values: $c_i = c_{if} = 1$, $f_i = 0$, $\eta = 2$, $\gamma = 0.1$, $\sigma = 1$.

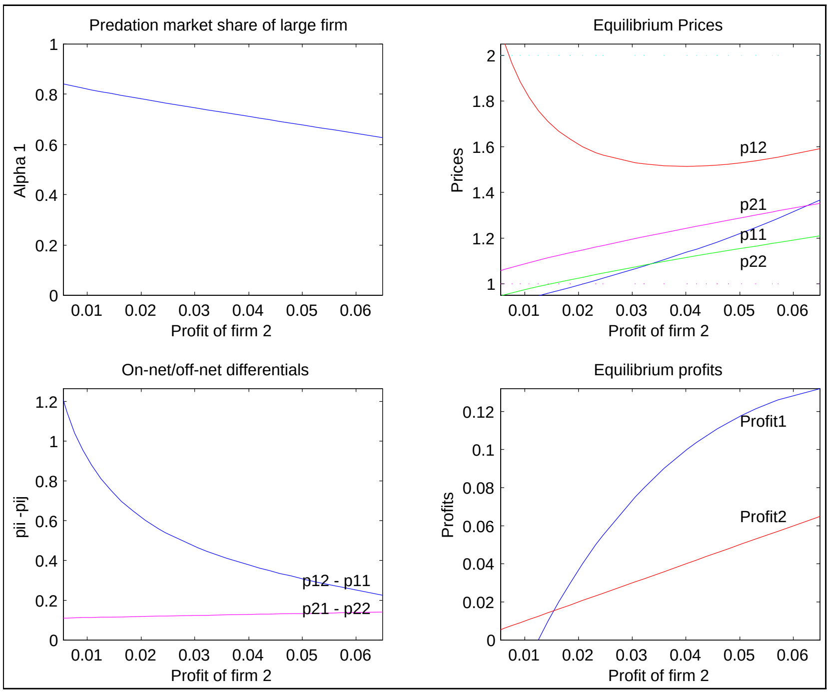

Figure 2: Predation Equilibria with linear tariffs as profits of firm 2 are decreased from the Nash equilibrium level. Parameter values: $c_i = c_{if} = 1$, $f_i = 0$, $\eta = 2$, $\gamma = 0.1$, $\sigma = 1$, $A = 0.2$.

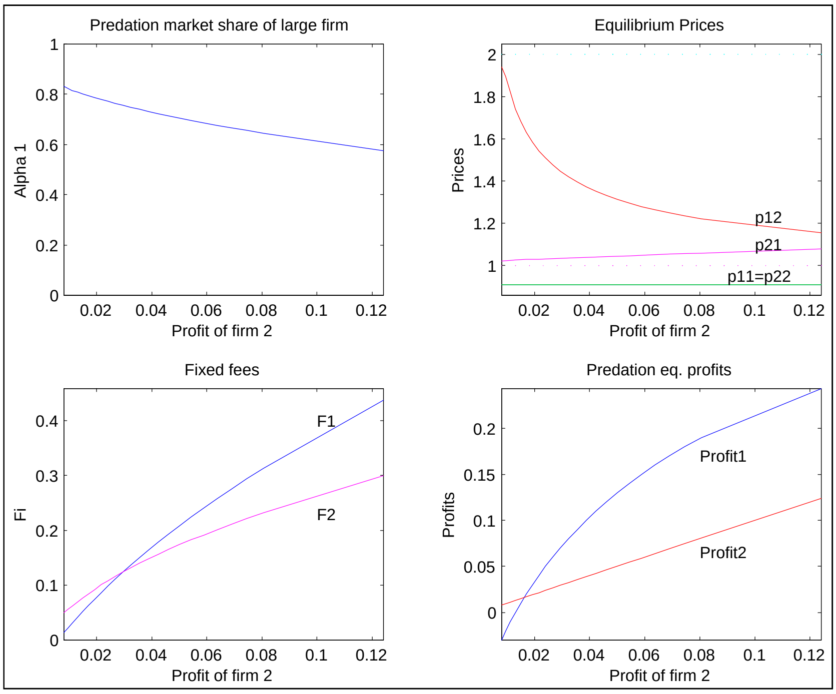

Figure 3: Predation Equilibria with two-part tariffs as profits of firm 2 are decreased from the Nash equilibrium level. Parameter values: $c_i = c_{if} = 1$, $f_i = 0$, $\eta = 2$, $\gamma = 0.1$, $\sigma = 1$, $A = 0.2$.

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