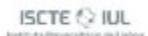


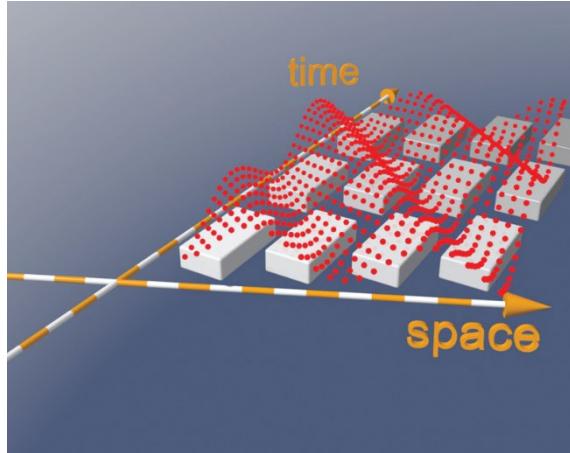
Engineering Arbitrary Metamaterial Responses with Spacetime Modulation

17º Congresso do Comité Português da URSI-ANACOM
"Materiais inteligentes para a radiociência"

Lisbon, 24 November 2024



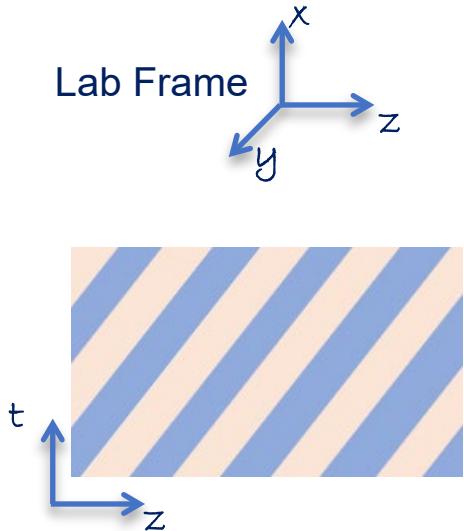
Spacetime Metamaterials



O.J.F. Martin, Advanced Photonics vol. 1, p. 050501 (2019)

- E. Galiffi, R. Tirole, S. Yin, H. Li, S. Vezzoli, P. A. Huidrobo, M. G. Silveirinha, R. Sapienza, A. Alù and J. B. Pendry, Advanced Photonics 4, 014002 (2022).
- F. Biancalana, A. Amann, A. V. Uskov, and E. P. O'Reilly, Phys. Rev. E 75, 046607 (2007).
- Z. Yu and S. Fan, Nature Photonics 3, 91 (2009).
- Z. Deck-Léger, N. Chamanara, M. Skorobogatiy, M. G. Silveirinha, and C. Caloz, Advanced Photonics 1, 1 – 26 (2019).
- X. Wang, A. Díaz-Rubio, H. Li, S. A. Tretyakov, and A. Alù, Phys. Rev. Applied 13, 044040 (2020).
- V. Pacheco-Peña, N. Engheta, “Temporal aiming”, Light: Science & Applications 9, 1-12, (2020).

Travelling Wave Modulation

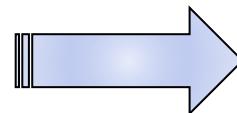


Materials response

$$\varepsilon(\mathbf{r}, t) = \varepsilon(\mathbf{r} - \mathbf{v}_0 t)$$

$$\mu(\mathbf{r}, t) = \mu(\mathbf{r} - \mathbf{v}_0 t)$$

Galilean Transformation

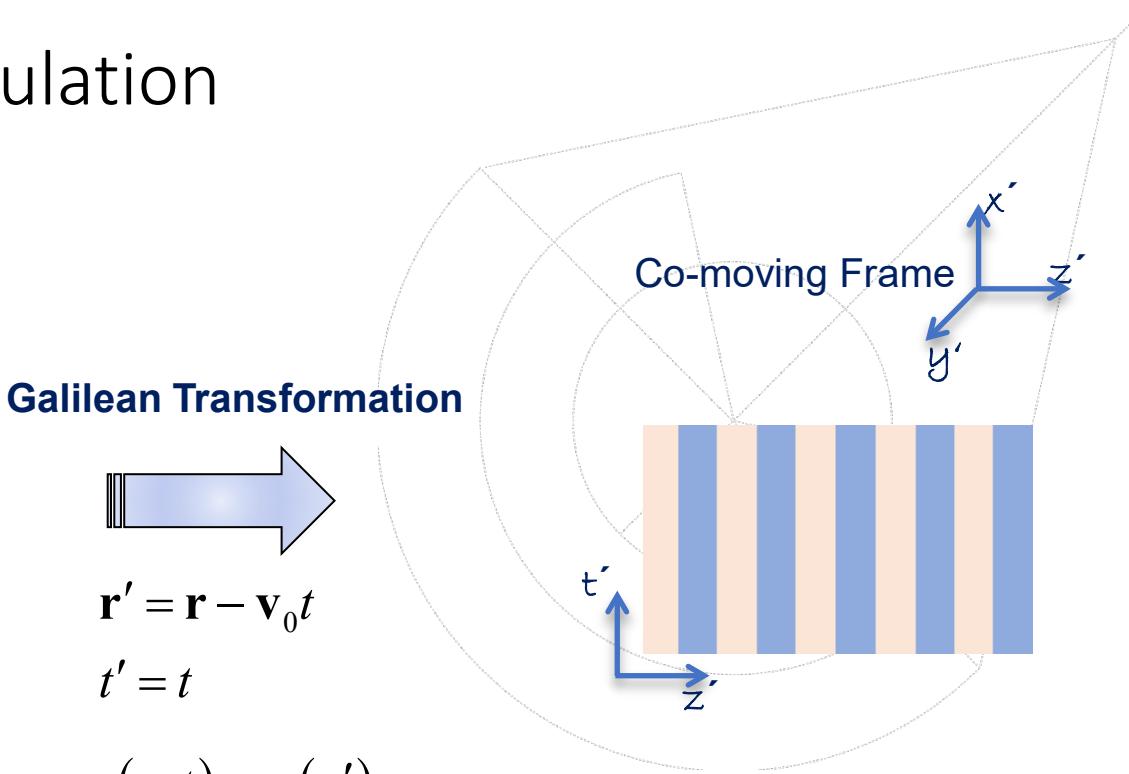


$$\mathbf{r}' = \mathbf{r} - \mathbf{v}_0 t$$

$$t' = t$$

$$\varepsilon(\mathbf{r}, t) = \varepsilon(\mathbf{r}')$$

$$\mu(\mathbf{r}, t) = \mu(\mathbf{r}')$$



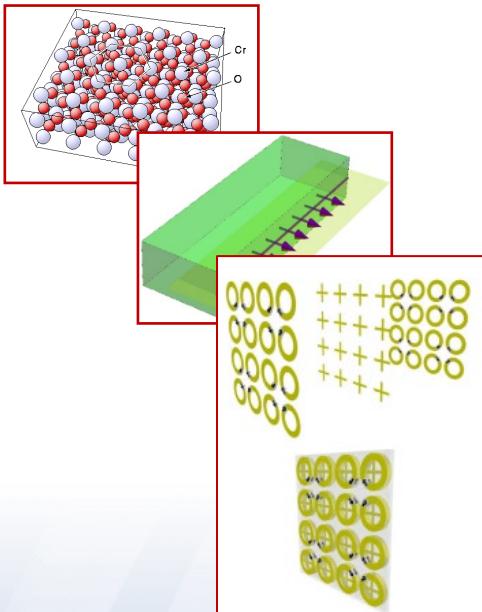
Tellegen Type-Response

Nonreciprocal and isotropic response

Tellegen constitutive relations

$$\mathbf{D} = \epsilon_0 \epsilon \mathbf{E} + c^{-1} \kappa \mathbf{H}$$

$$\mathbf{B} = \mu_0 \mu \mathbf{H} + c^{-1} \kappa \mathbf{E}$$



The chromium oxide Cr_2O_3 is an uniaxial Tellegen material

D. N. Astrov, Sov Phys JETP, vol. 13, 729 (1961).

Artificial Tellegen particle

S. A. Tretyakov, S. I. Maslovski, I. S. Nefedov, A. J. Viitanen, P. A. Belov and A. Sanmartin, Electromagnetics 23, 665 (2003).

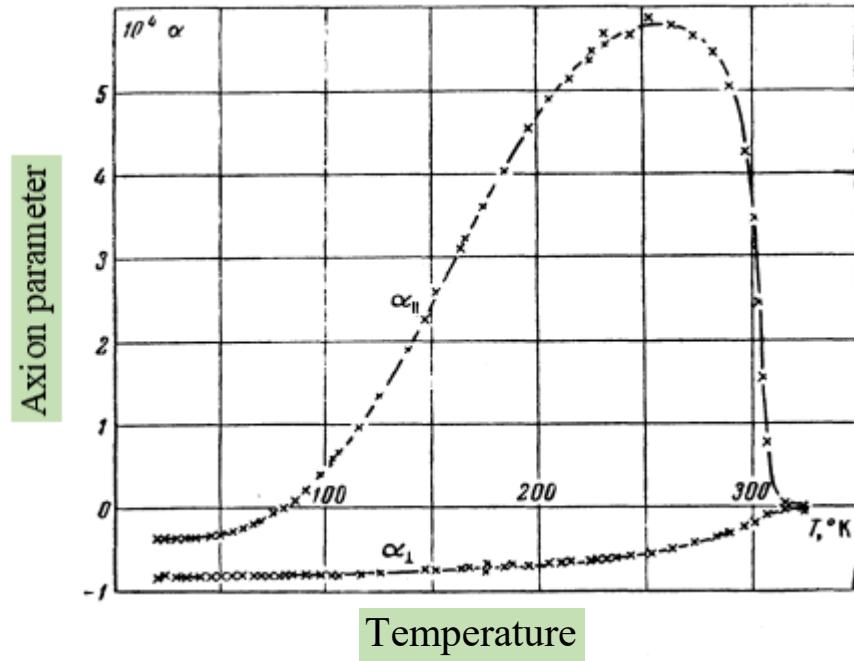
Magnet-free bianisotropic Tellegen responses

Y. Ra'di and A. Grbic, Phys. Rev. B 94, 195432 (2016).

Topological insulators may have a Tellegen-type electromagnetic response

V. Dzjom, et al., Nat. Commun. 8, 15197 (2017).

Tellegen Type-Response in Natural Materials



D. N. Astrov, " Magnetoelectric effect in **chromium oxide**," Sov Phys JETP, vol. 13, 729 (1961).

Homogenization of anisotropic spacetime systems

Homogenization of Spacetime Systems

Lab Frame

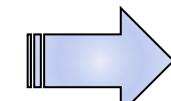
$$\begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix} = \begin{pmatrix} \bar{\varepsilon}(z - v_0 t) & \mathbf{0} \\ \mathbf{0} & \bar{\mu}(z - v_0 t) \end{pmatrix} \cdot \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}$$

$\underbrace{\hspace{10em}}_{\mathbf{M}(z - v_0 t)}$

Anisotropic spacetime tensors

$$\bar{\varepsilon}(z - v_0 t) \quad \bar{\mu}(z - v_0 t)$$

Galilean transformation



$$\begin{aligned} \mathbf{r}' &= \mathbf{r} - \mathbf{v}_0 t \\ t' &= t \end{aligned}$$

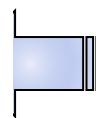
Co-moving Frame

Dynamics in the **co-moving frame**:

$$\begin{pmatrix} \mathbf{D}' \\ \mathbf{B}' \end{pmatrix} = \mathbf{M}'(z') \cdot \begin{pmatrix} \mathbf{E}' \\ \mathbf{H}' \end{pmatrix}$$

Bi-anisotropic medium

$$\mathbf{M}' = \mathbf{M} \cdot [\mathbf{1}_{6 \times 6} + \mathbf{V} \cdot \mathbf{M}]^{-1}$$

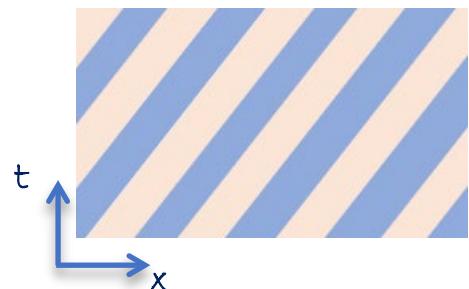


nsformat

P. A. Huidobro, M. G. Silveirinha, E. Galiffi, J.B. Pendry, Phys. Rev. App., 16, 014044, 2021.

Effective Moving Medium in the Long Wavelength Limit

Spacetime modulated crystal



Isotropic (scalar) material tensors

$$\varepsilon(\mathbf{r}, t) = \varepsilon(\mathbf{r} - \mathbf{v}_0 t)$$

$$\mu(\mathbf{r}, t) = \mu(\mathbf{r} - \mathbf{v}_0 t)$$

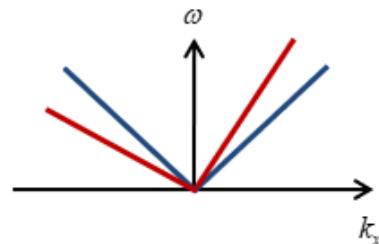
Equivalent moving medium:

uniaxial, non magneto-electric, moving at speed v_D

$$\boldsymbol{\epsilon}_{\text{eq}} = \begin{bmatrix} \epsilon_{\text{eq},||} & 0 & 0 \\ 0 & \epsilon_{\text{eq}} & 0 \\ 0 & 0 & \epsilon_{\text{eq}} \end{bmatrix}; \quad \boldsymbol{\mu}_{\text{eq}} = \begin{bmatrix} \mu_{\text{eq},||} & 0 & 0 \\ 0 & \mu_{\text{eq}} & 0 \\ 0 & 0 & \mu_{\text{eq}} \end{bmatrix}.$$



Dispersion diagram in the long wavelength limit



Paloma A. Huidobro, Emanuele Galiffi, Sébastien Guenneau, Richard V. Craster, and J. B. Pendry, PNAS, 116 (50) 24943-24948, 2019.

P. A. Huidobro, M. G. Silveirinha, E. Galiffi, J.B. Pendry, Phys. Rev. App., 16, 014044, 2021.

Anisotropic Spacetime Crystal

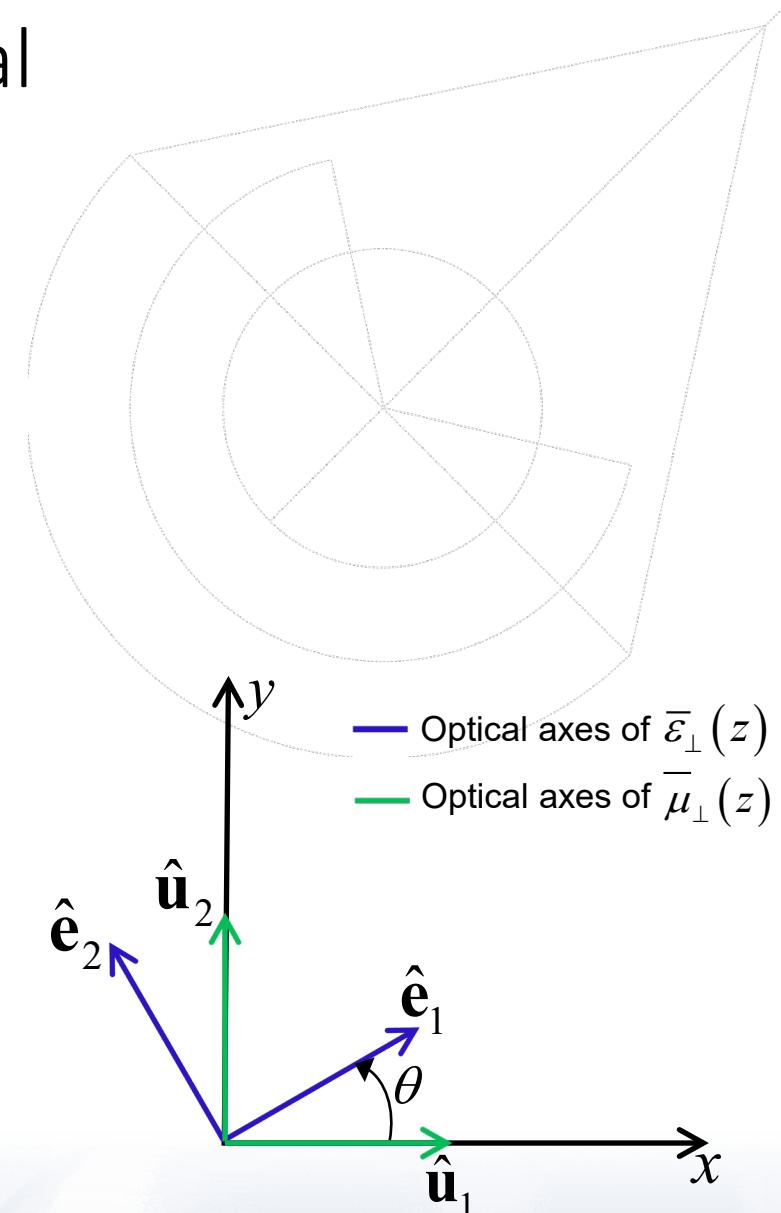
Anisotropic material parameters

$$\bar{\mu}_{\perp}(z,t) = \mu_1(z') \hat{\mathbf{u}}_1 \otimes \hat{\mathbf{u}}_1 + \mu_2(z') \hat{\mathbf{u}}_2 \otimes \hat{\mathbf{u}}_2$$

$$\bar{\epsilon}_{\perp}(z,t) = \epsilon_1(z') \hat{\mathbf{e}}_1 \otimes \hat{\mathbf{e}}_1 + \epsilon_2(z') \hat{\mathbf{e}}_2 \otimes \hat{\mathbf{e}}_2$$

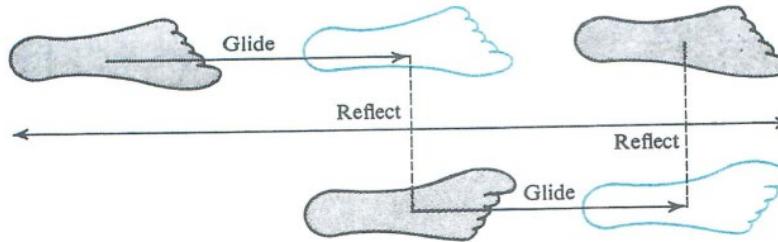
$$z' = z - vt$$

$\theta \rightarrow$ Offset angle between the permeability and permittivity axes



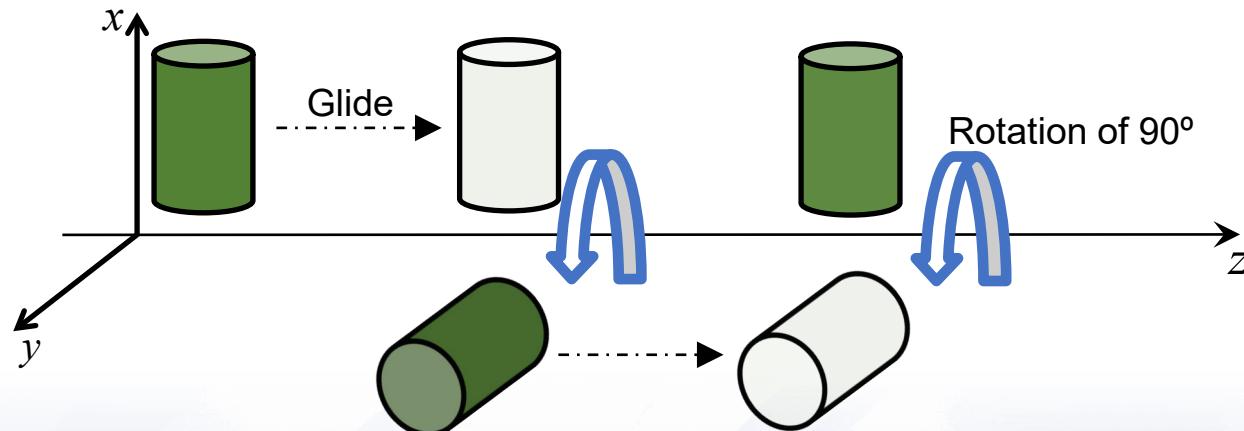
Glide Symmetry

Generalized glide symmetry: translation + **reflection**



<https://tinyurl.com/2trvw295>

Glide symmetry: translation + **rotation of 90°**



Effective Moving-Tellegen Response for Crystals with Glide Symmetry

Constitutive relations for fields in the **xoy plane**

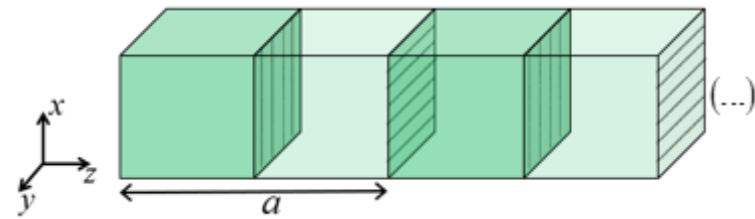
$$\langle \mathbf{D} \rangle = \epsilon_0 \epsilon_{\text{ef},\perp} \langle \mathbf{E} \rangle + \frac{1}{c} \kappa_{\text{ef}} \langle \mathbf{H} \rangle - \frac{1}{c} \xi_{\text{ef}} \hat{\mathbf{z}} \times \langle \mathbf{H} \rangle$$

$$\langle \mathbf{B} \rangle = \mu_0 \mu_{\text{ef},\perp} \langle \mathbf{H} \rangle + \frac{1}{c} \kappa_{\text{ef}} \langle \mathbf{E} \rangle + \frac{1}{c} \xi_{\text{ef}} \hat{\mathbf{z}} \times \langle \mathbf{E} \rangle$$

κ_{ef} : (axion) Tellegen parameter

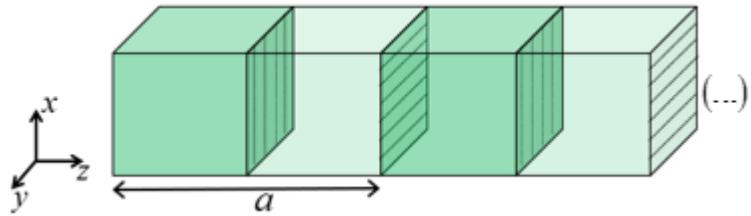
ξ_{ef} : moving medium piece

Two-Phase Anisotropic Spacetime Crystal



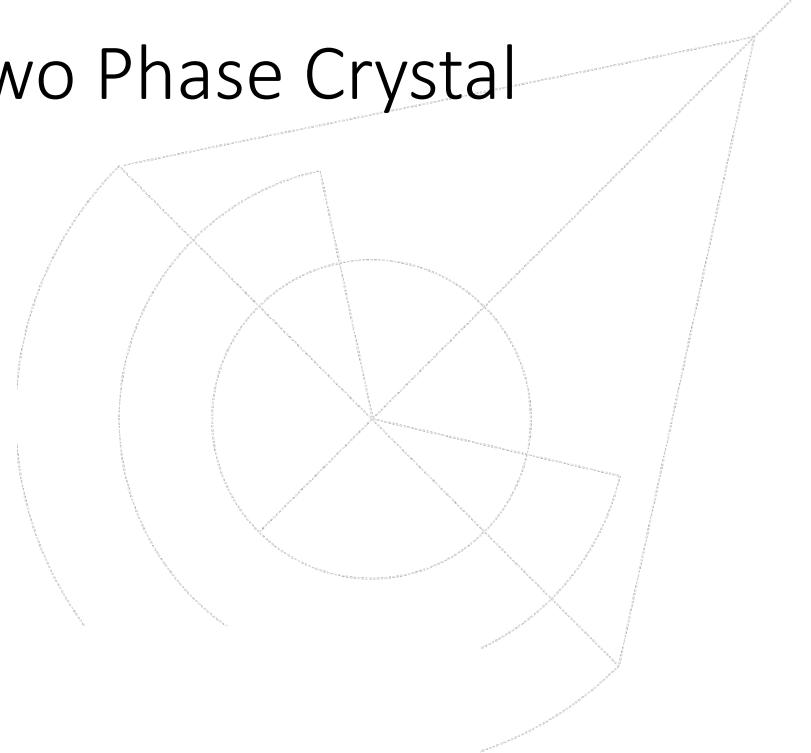
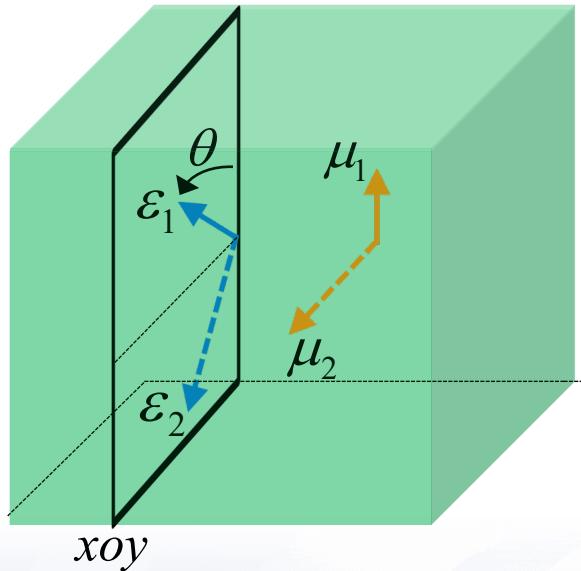
Glide-Rotation Symmetry for a Two Phase Crystal

Two-phase spacetime crystal

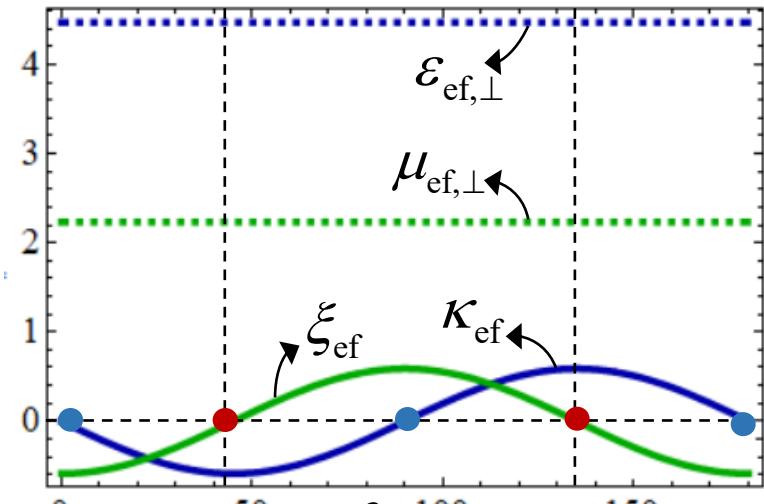
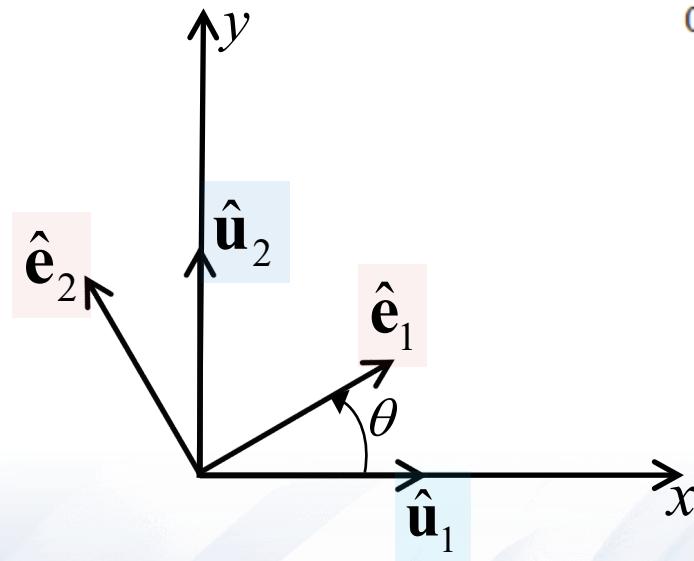


$$\bar{\varepsilon}_A \quad \bar{\mu}_A$$

$$\bar{\varepsilon}_B \quad \bar{\mu}_B$$



Effective Response



Effective Moving-Tellegen Response

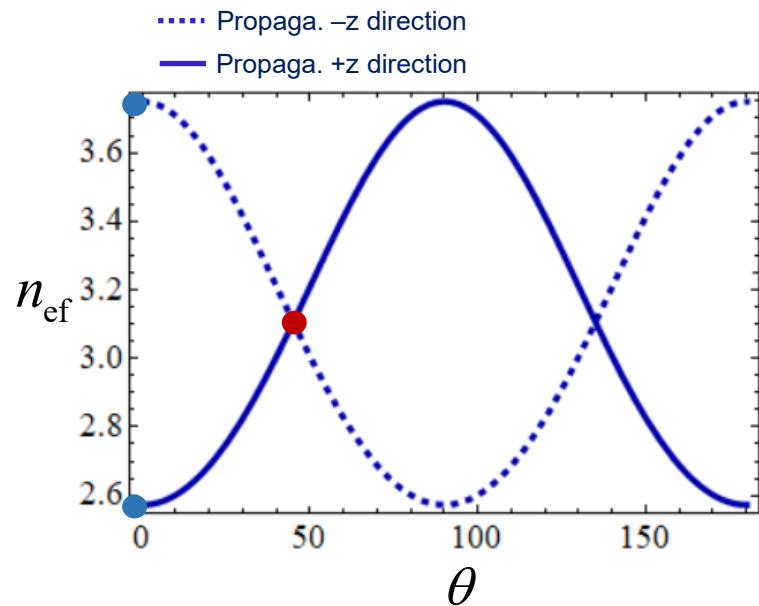
$$\varepsilon_{\text{ef},\perp} = \frac{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2 (\mu_1 + \mu_2) v^2}{2 - \frac{1}{2}(\varepsilon_1 + \varepsilon_2)(\mu_1 + \mu_2)v^2}$$

$$\kappa_{\text{ef}} = -\frac{v}{2} \frac{(\varepsilon_1 - \varepsilon_2)(\mu_1 - \mu_2) \sin 2\theta}{2 - \frac{1}{2}(\varepsilon_1 + \varepsilon_2)(\mu_1 + \mu_2)v^2}$$

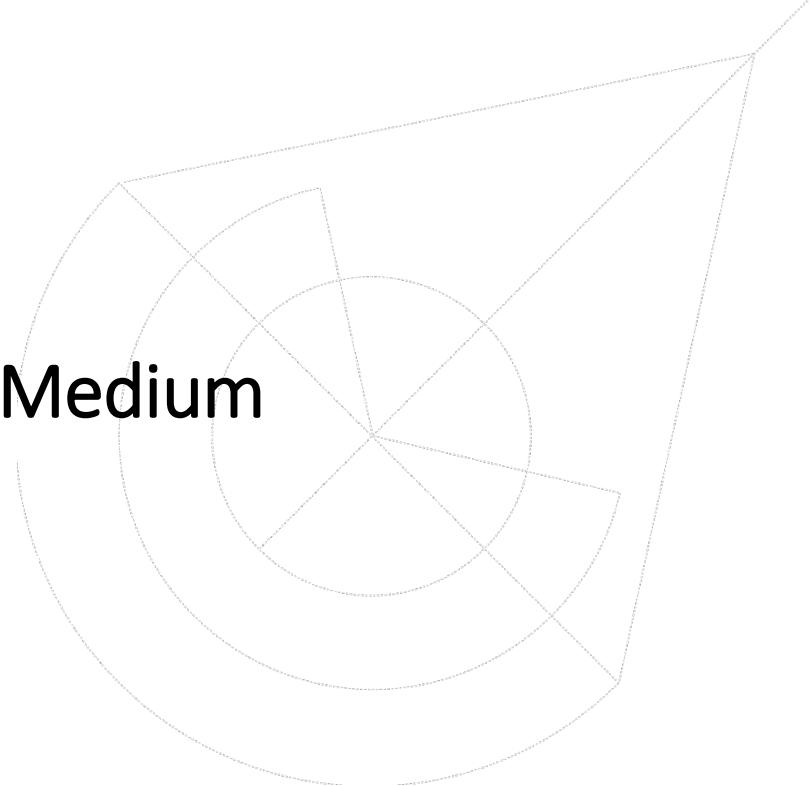
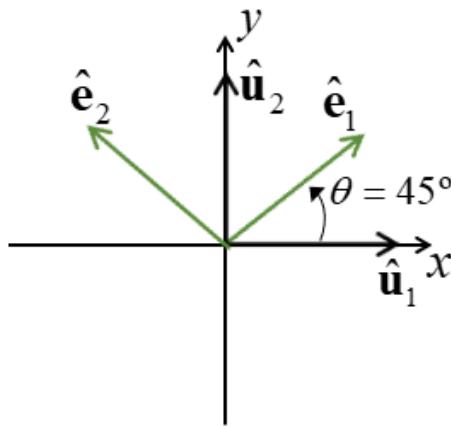
$$\mu_{\text{ef},\perp} = \frac{\mu_1 + \mu_2 - \mu_1 \mu_2 (\varepsilon_1 + \varepsilon_2) v^2}{2 - \frac{1}{2}(\varepsilon_1 + \varepsilon_2)(\mu_1 + \mu_2)v^2}$$

$$\xi_{\text{ef}} = -\frac{v}{2} \frac{(\varepsilon_1 - \varepsilon_2)(\mu_1 - \mu_2) \cos 2\theta}{2 - \frac{1}{2}(\varepsilon_1 + \varepsilon_2)(\mu_1 + \mu_2)v^2}$$

Effective Moving-Tellegen Response



Effective Tellegen Medium



Effective Tellegen Medium

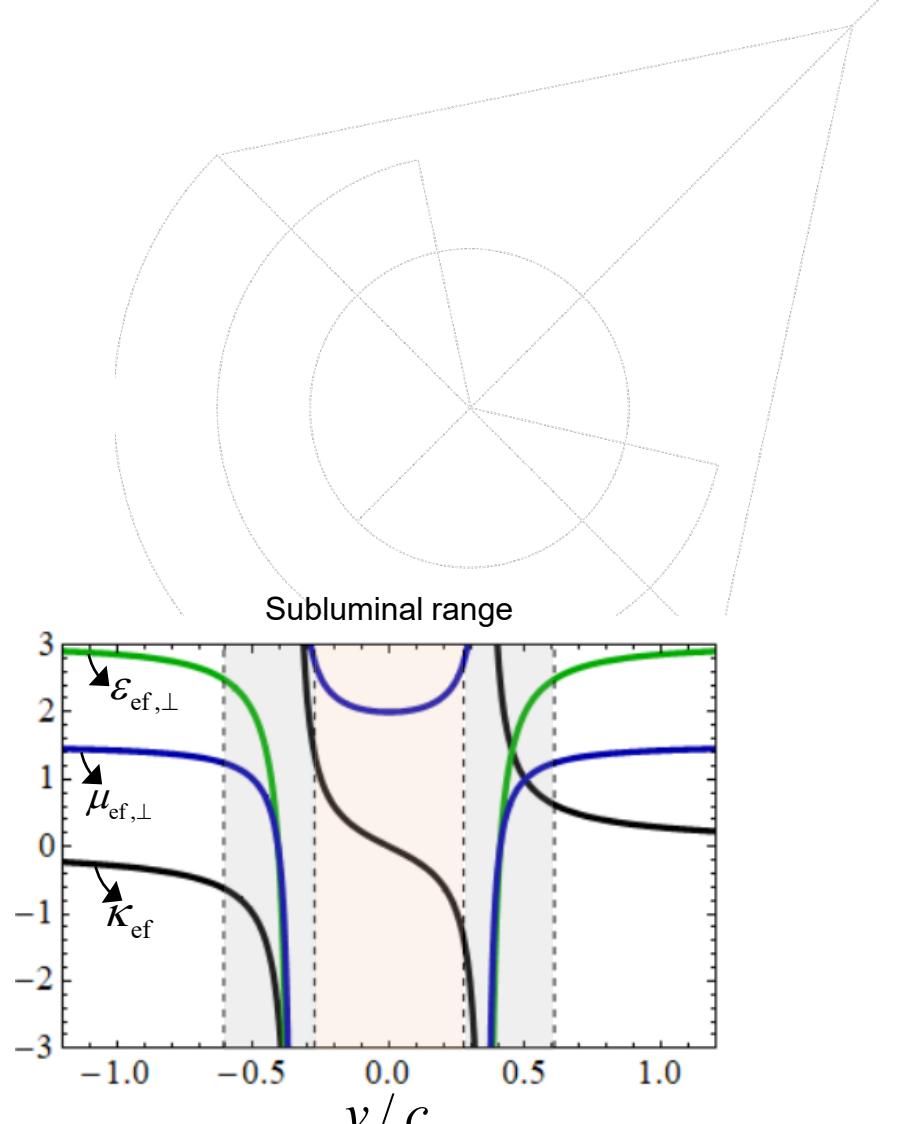
Tellegen constitutive relations

$$\langle \mathbf{D} \rangle = \epsilon_0 \epsilon_{\text{ef},\perp} \langle \mathbf{E} \rangle + c^{-1} \kappa_{\text{ef}} \langle \mathbf{H} \rangle$$

$$\langle \mathbf{H} \rangle = \mu_0 \mu_{\text{ef},\perp} \langle \mathbf{H} \rangle + c^{-1} \kappa_{\text{ef}} \langle \mathbf{E} \rangle$$

Refractive index

$$n_{\text{ef}} = \sqrt{\epsilon_{\text{ef},\perp} \mu_{\text{ef},\perp} - \kappa_{\text{ef}}^2}$$



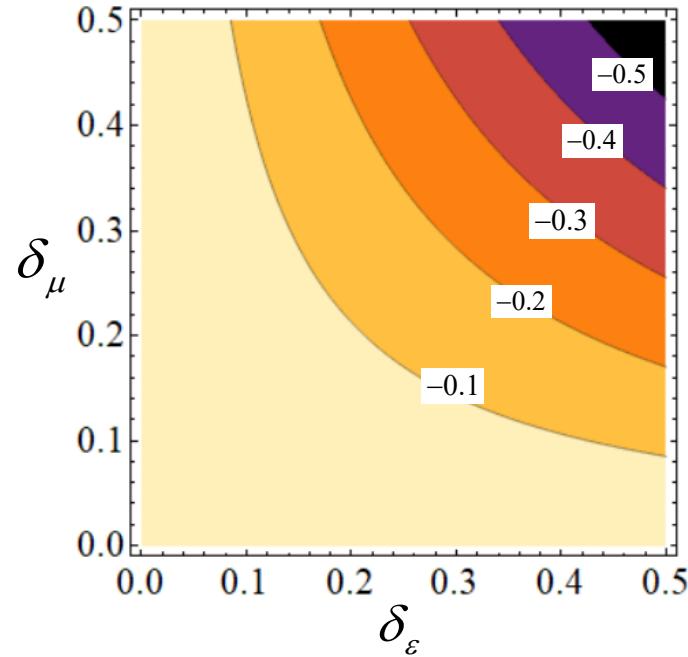
$$\begin{array}{ll} \epsilon_1 = 6 & \mu_1 = 3 \\ \epsilon_2 = 2 & \mu_2 = 1 \end{array}$$

Tellegen Parameter

$$\begin{aligned}\varepsilon_1 &= \varepsilon_M (1 + \delta_\varepsilon), & \varepsilon_2 &= \varepsilon_M (1 - \delta_\varepsilon) \\ \mu_1 &= \mu_M (1 + \delta_\mu), & \mu_2 &= \mu_M (1 - \delta_\mu)\end{aligned}$$

Effective Tellegen parameter

$$K_{\text{ef}} = -\delta_\varepsilon \delta_\mu \frac{\varepsilon_M \mu_M v}{1 - \varepsilon_M \mu_M v^2}$$



Giant Tellegen parameter

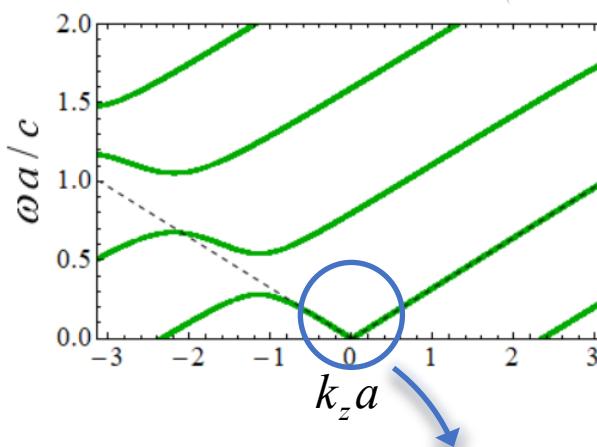
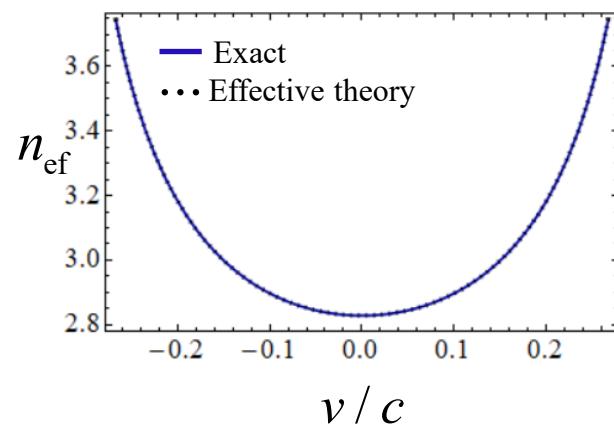
$$K_{\text{ef}} \approx -0.59$$

$$\delta_\varepsilon = \delta_\mu = 0.5$$

$$\begin{aligned}\varepsilon_M &= 4 \\ \mu_M &= 2 \\ v / c &= 0.2\end{aligned}$$

Comparison Between the Exact and the Effective Response

$$n_{\text{ef}} = \sqrt{\varepsilon_{\text{ef},\perp} \mu_{\text{ef},\perp} - \kappa_{\text{ef}}^2}$$

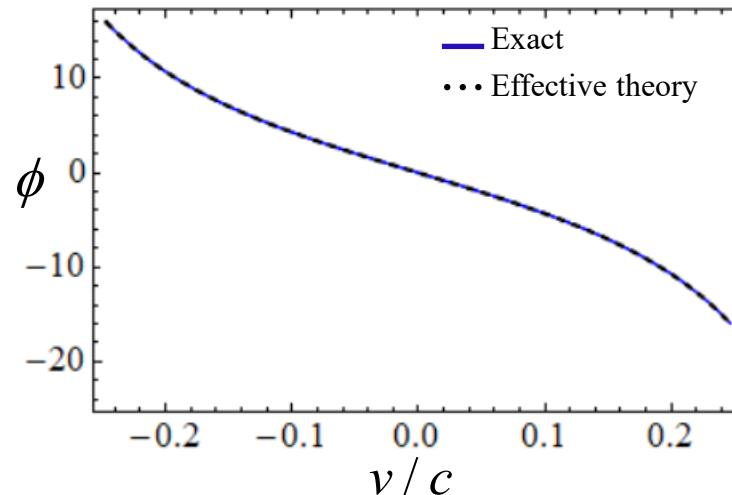
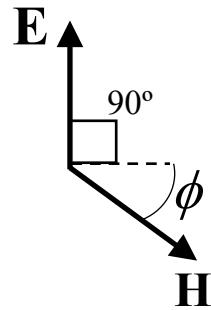


Effective response in the quasi-static limit

$$\begin{array}{lll} v/c = 0.2 & \varepsilon_1 = 6 & \mu_1 = 3 \\ \varepsilon_2 = 2 & \mu_2 = 1 \end{array}$$

Offset Angle Between the Magnetic and Electric Fields

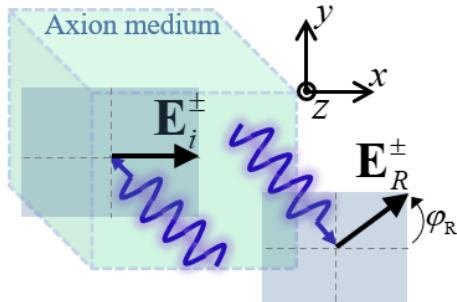
$$\phi = \arcsin\left(\frac{\kappa_{\text{ef}}}{\sqrt{\epsilon_{\text{ef},\perp} \mu_{\text{ef},\perp}}}\right)$$



$$\begin{array}{ll} \epsilon_1 = 6 & \mu_1 = 3 \\ \epsilon_2 = 2 & \mu_2 = 1 \end{array}$$

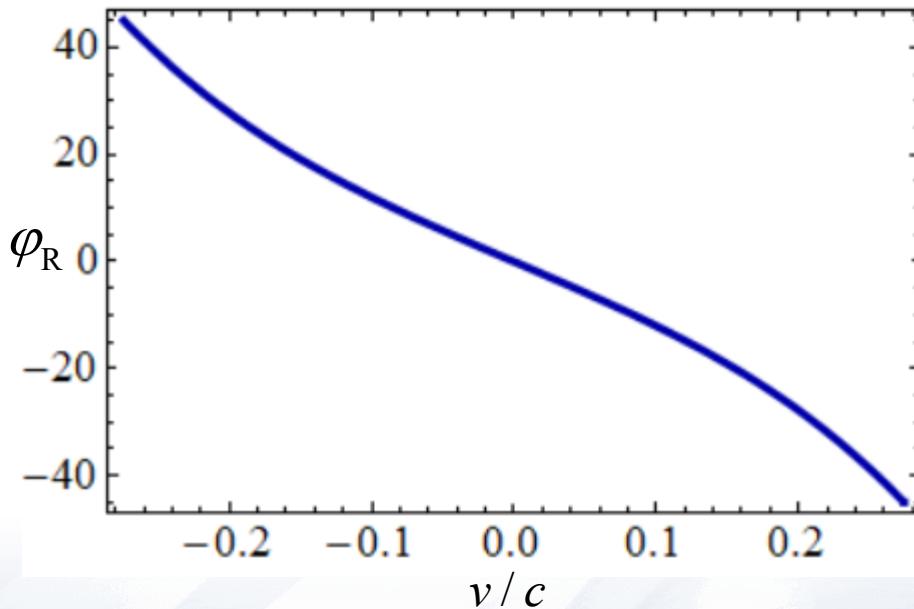
Nonreciprocal Effects: Polarization Rotation

air-synthetic axion medium interface



Angle of rotation

$$\varphi_R = \arctan\left(\frac{2\kappa_{\text{ef}}}{\epsilon_{\text{ef},\perp} - \mu_{\text{ef},\perp}}\right)$$



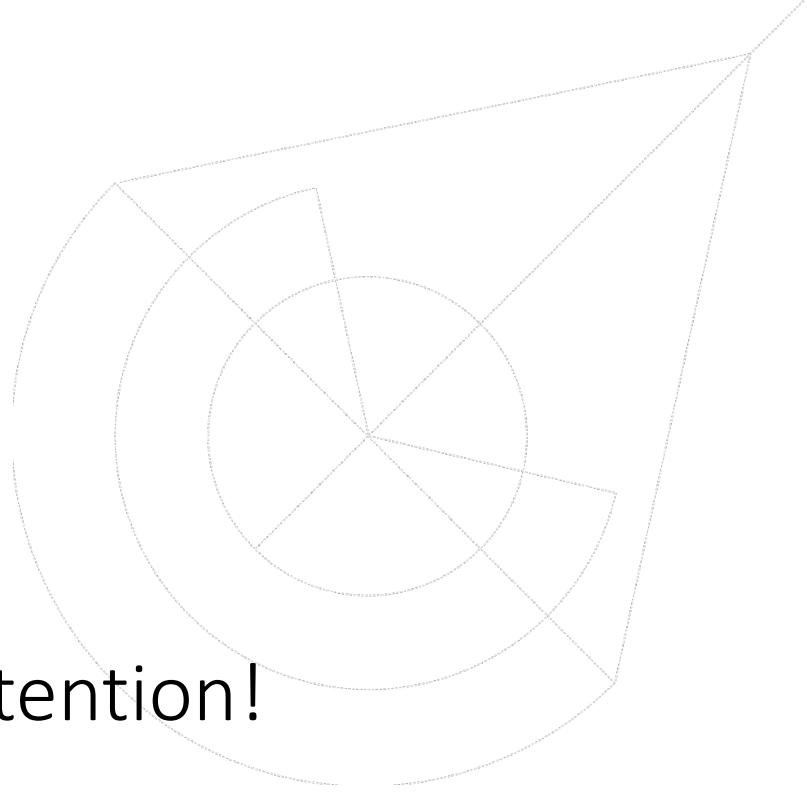
Near the subluminal threshold

$$\varphi_R \approx -27.76^\circ$$

$$\begin{array}{ll} \epsilon_1 = 6 & \mu_1 = 3 \\ \epsilon_2 = 2 & \mu_2 = 1 \end{array}$$

Summary

- It was demonstrated that spacetime modulations provide an exciting path to sculpt the nonreciprocal response in the long wavelength limit.
- While for isotropic materials the effective medium behaves always as a moving material, **crystals made of anisotropic materials** offer increased design flexibility, and in particular allow the synthesis of **the (axion) Tellegen response**.
- It was shown that when the spacetime crystal has a **glide-rotation symmetry** its effective response in the xoy plane is polarization independent, and the effective material behaves as a **uniaxial moving-Tellegen medium**.
- Even for very moderate modulation speeds the axion parameter can be several orders of magnitude larger than what is observed in any natural material.



Thank you for your attention!